ARKIV FÖR FYSIK Band 13 nr 35

Communicated 9 April 1958 by MANNE SIEGBAHN and BENGT EDLÉN

An investigation of the positive point streamer corona

Part II. The field strength necessary for the streamer propagation

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With 19 figures in the text

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SUMMARY

The field strength necessary for the propagation of the positive point streamers, E', is studied in 11 different gas mixtures. This field strength is highly dependent on the composition of the gas. As an example we have: E' is lower than 3 kV/cm in $N_2 + A$ mixtures but is about 14 kV/cm in oxygen (atmospheric pressure). The elementary processes in the discharge that can give rise to the great difference in E' are discussed and all but one must be rejected in a qualitative study. It is only the formation of negative ions that increases in the same way as E'. A quantitative study supports the hypothesis that the increase in E' is caused by the formation of negative ions in the streamer.

List of symbols

a, b, d	Distances, defined where they are used.
e	Electronic charge.
E	Field strength.
E'	Critical field strength, experimentally measured.
E_1	Field strength due to a positive space charge.
E_2	Field strength due to the increase of the negative space charge that is caused by the formation of negative ions.
E_3	Field strength necessary to increase the drift velocity of the electrons so that the number of negative charges per cm in the streamer is the same when negative ions are formed as when no negative ions are formed.
k	Mobility of the electrons at atmospheric pressure.
L_0, L	Distances, defined where they are used.
l	Distance to the streamer tip.
n_0	Number of ion pairs produced per cm of the streamer.
N	Number of electrons per cm in the streamer when no negative ions are formed.
n_+	$n_+ = n_0 - N$ = excess number of positive charges.
n_e	Number of electrons per cm in the streamer when negative ions are formed.
n_-	Number of negative ions per cm in the streamer.
$N(\eta)$	$N(\eta) = n_- + n_e$ = total number of negative charges per cm of the streamer.
p	Pressure.
$p(O_2)$	Partial pressure of oxygen.
p_x	Partial pressure of the gas mixed to the nitrogen (Fig. 7).
q	Charge per unit length of a cylindrical space charge.
q_0	Total charge produced per cm of the streamer.
q_1	Positive charge per cm of the streamer, $q_1 = q_0 \dfrac{v_e}{v_e + v_t}$.
q_2	Negative charge per cm of the streamer, $q_2 = q_0 \dfrac{v_e}{v_e + v_t}$.
R	The radius of the streamer.
r	Radius.
s	$s = \int_0^l \dfrac{v_e(l)}{v_e(l) + v_t} \cdot dl.$

t	Time.
T	Time, defined where it is used.
V	The voltage of the point.
V_Q	The voltage difference between the plane P and the plane Q (Fig. 1).
v_e	Drift velocity of the electrons (constant, independent of l).
$v_e(l)$	Drift velocity of the electrons at the distance l from the streamer tip when negative ions are formed.
$v_e'(l)$	Drift velocity of the electrons at the distance l from the streamer tip when *no* negative ions are formed.
$v_e(x)$	Drift velocity of the electrons at the distance x from the streamer tip.
v_t	Velocity of the streamer tip $= 1.3 \times 10^7$ cm/sec.
X	$X = E_1/q_1$ (Eq. 13 and Fig. 15).
x	Distance, defined where it is used.
y	$y = \dfrac{v_e'(l)}{v_e'(l) + v_t}$.
z	$z = \dfrac{v_e(l)}{v_e(l) + v_t}$.
α	The first Townsend ionization coefficient.
ϑ	Angle.
η	Attachement coefficient.
$\varkappa_0$	Permittivity of free space.

Introduction

In an earlier paper [1] it was shown that the combined continuous and streamer corona (positive point) is very sensitive to the amount of water vapour in the air and therefore can be used as a fast hygrometer. Furthermore, this type of discharge has been observed at water drops hanging from a metal rod which is charged to a high potential. Consequently, it must highly affect the corona losses at high tension lines.

A detailed description of the discharge was given in Part I of this investigation [2]. As the same experimental arrangements and the same point shape are used as in Part I only a short summary will be given here.

The investigators of the pre-onset streamer discharge [3–15] used hemispherical points and did not obtain the streamer discharge as a self-sustaining discharge. In this investigation conical points are used (see Part I), and the streamers are obtained in a self-sustaining discharge. In Part I [2] the time distribution of the streamers was measured and it was found that after a streamer has been formed there is a dead time during which no new streamer can be formed. It was shown that this dead time is caused by the arrival of negative ions at the point. These negative ions are formed in the streamer by attachment of electrons to gas molecules. The negative ions formed furthest from the point determine the dead time, for the dead time is equal to the time needed for these ions to reach the point.

In this part of the investigation the field strength necessary for the streamer propagation has been measured in a number of different gas mixtures. This field strength is very much dependent on the composition of the gas in the corona gap. Many of the elementary processes are the same in breakdown streamers as in the streamers from a positive point. Consequently, it is of great importance to find out which of these processes exert a great influence on the field necessary for the streamer propagation. A thorough discussion of this is given below.

As we know that the negative ions formed in the streamer prevent the streamer formation and cause a dead time, it is easy to find which factors determine the length of this dead time. These factors must be the drift velocity of the negative ions and the length of the streamers. The drift velocity is dependent on the field strength and the composition and pressure of the gas. The length of the streamers is above all determined by the field strength and the composition and pressure of the gas, but also the shape and arrangement of the electrodes and some other factors exert a certain influence. In point-to-plane geometry an increase in the length of the streamers greatly affects the length of the dead time. The field strength decreases rapidly with increasing distance to the point; the greater the distance to the point, the longer the time required for the ions to move a certain distance.

A. EXPERIMENTAL PART

1. The critical field strength

If the voltage pulses across a resistance in series with the streamer discharge are studied on an oscilloscope screen we observe that all the voltage pulses have about the same height (Part I, Fig. 4). Of course it is supposed that no factor that can affect the streamer formation is altered. On taking this into consideration, it is plausible to suppose that about the same number of electrons and positive ions are produced in each streamer and that all the streamers have about the same length. Consequently, it is to be expected that the streamers cannot propagate in a space where this field strength is lower than a certain value. Of course this value is dependent on the pressure and composition of the gas and a number of other factors. A close investigation of this dependence can hardly be made in the highly non-uniform field in point-to-plane arrangement, where it is almost impossible to determine the field strength. However, the arrangement shown in Fig. 1 can be used.

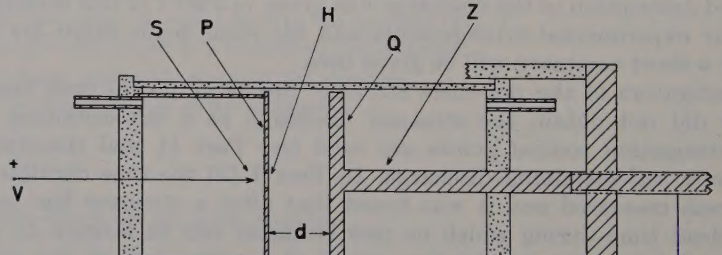

Fig. 1. The arrangement used for measurement of the critical field strength. The streamers are formed at *S*, pass through the hole *H* and reach the plane *Q*. The distance *d* can be varied by turning the threaded rod *Z*.

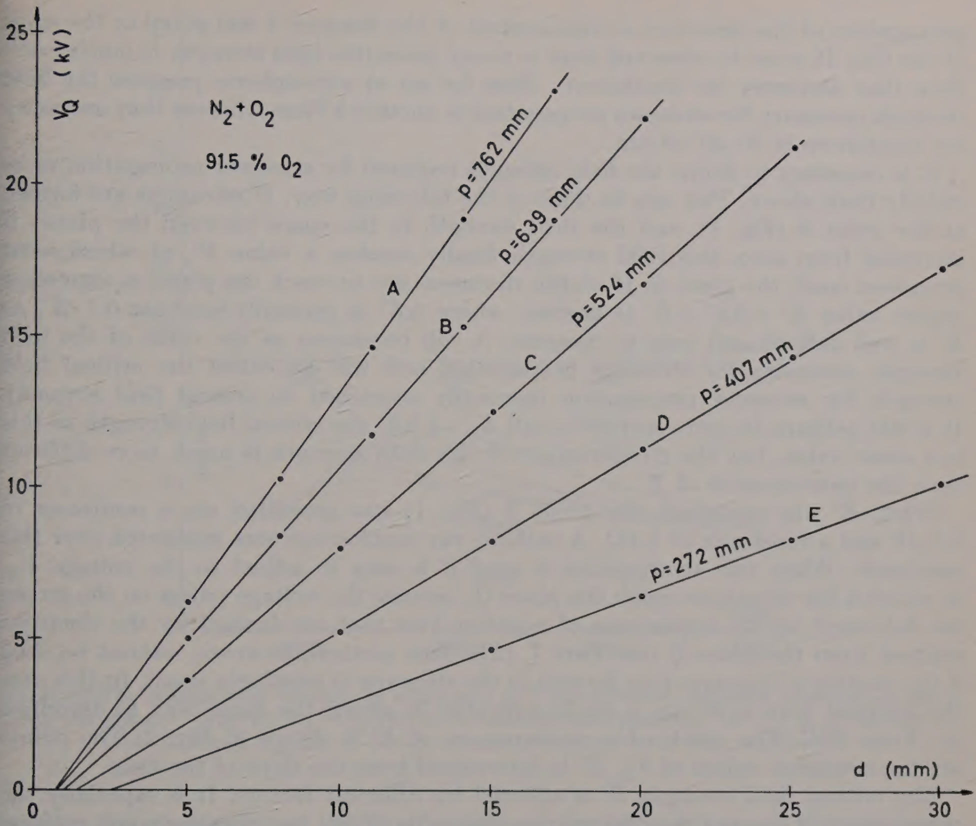

Fig. 2. The potential difference V_Q between the planes P and Q as a function of d. V_Q is the potential difference when some of the streamers reach the plane Q. E′ is equal to the slope of the lines.

The arrangement shown in Fig. 1 consists of the three electrodes S, P, and Q, which are fitted in a container made of perspex. The streamers are formed at the point S, the shape of which is the same as that shown in Fig. 2, Part I. The electrode P is a plate in which a hole H has been drilled opposite to the point. The third electrode Q consists of a plate placed parallel to the plane of P. The distance between the two planes is d. The plate Q is supported by the metal rod Z, the outer end of which is threaded. The distance d can be changed by turning the rod.

If the distance between the point S and the plane P is about the same as the length of the streamers—that is, a few millimetres, and if the field strength in the space between the planes is sufficiently high, the streamers pass through the hole H and do not stop until they reach the plane Q. Streamers of different length can be obtained by varying d. Since the field strength is not uniform in the neighbourhood of the hole, the field strength necessary for streamer propagation must be determined by plotting V_Q as a function of d, where V_Q is the voltage difference between the two plates P and Q. In all measurements that have been carried out V_Q has been found to be a linear function of d. Consequently, the field strength necessary for the propagation of the streamers is independent of the distance d and equal to the slope of the line. It must be observed that in many cases this field strength is much lower than that necessary for breakdown. Thus for air at atmospheric pressure the field strength necessary for streamer propagation is about 4 kV/cm whereas that necessary for breakdown is 20–30 kV/cm.

It is necessary to define the field strength required for streamer propagation more strictly than above. This can be done in the following way. If streamers are formed at the point S (Fig. 1), and the field strength in the space between the planes is increased from zero, this field strength finally reaches a value E', at which some streamers reach the plane Q. If all the streamers are to reach the plane, a somewhat higher value $E' + \Delta E' = E''$ is needed, where $\Delta E'$ is generally less than $0.1 \cdot E'$. As E' is well defined and easy to measure, it will be chosen as the value of the field strength necessary for streamer propagation and will be called the critical field strength for streamer propagation (generally shortened to *critical field strength*). It would perhaps be more correct to call $E' + \frac{1}{2}\Delta E'$ the critical field strength as this is a mean value, but the measurement of this field strength is much more difficult than the measurement of E'.

When E' was measured, the point S (Fig. 1) was grounded via a condenser of 0.2 μF and a resistance of 1 kΩ. A cathode ray oscilloscope was connected over this resistance. When this arrangement is used it is easy to adjust to the voltage V_Q, at which a few streamers reach the plane Q, because the voltage pulses on the screen are deformed by the appearence of negative ions that are formed by the electrons emitted from the plane Q (see Part I, [2]). This method, however, cannot be used if the number of negative ions formed in the streamer is relatively small. In this case the simplest way is to use a photomultiplier in about the same way as described by Amin [12]. The result of a measurement of E' is shown in Fig. 2. The points are the measured values of V_Q. E' is determined from the slope of the lines.

The critical field strength E' is affected by different factors. It is especially the composition of the gas that determines the value of E', but also the point voltage, the point–plane distance, and the shape of the point; other geometrical factors also have a certain influence.

2. The dependence of E′ on the point voltage and the point–plane distance

When the critical field strength is measured with the arrangement described above, the point–plane distance (Fig. 1) must, of course, be adjusted so that the streamers reach the hole H, independent of the field strength in the space between the planes. If this were not the case, they would have to pass a space where the field strength could be lower than in the space between the planes. The relation between the critical field strength and the point voltage at different point-plane distances is shown in Fig. 3. The measurement was made in a mixture of N_2 and O_2 with 51.7 % O_2. The point was placed at the distances 0.5, 1.0, 1.5, and 2.0 mm from the plane P, which was about 0.4 mm thick in the centre. The diameter of the hole H was 3.0 mm.

The critical field strength is low at low and high values of the point voltage and has a maximum at a few hundred volts above onset. This shape of the curve is independent of the point–plane distance, if this distance is not too large. The distance 2.0 mm is in this case about the same as the length of the streamers with zero field

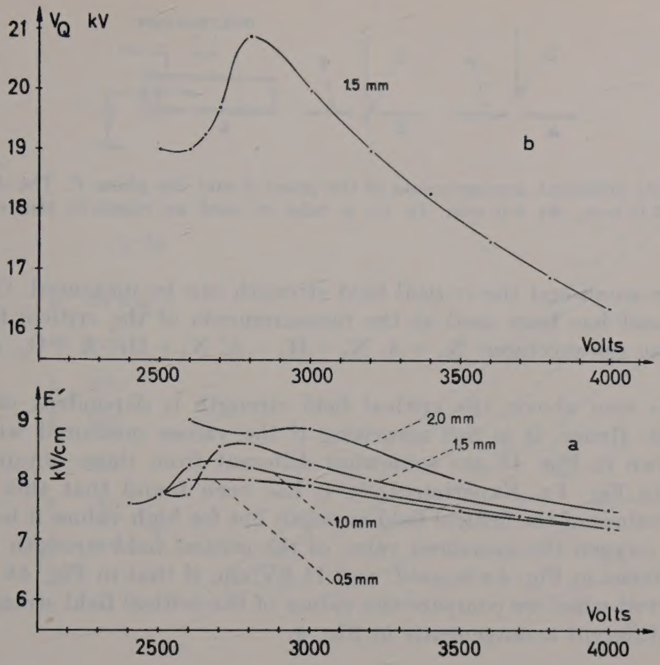

Fig. 3. (*a*) The critical field strength as a function of V for different distances between the point S and the plane P (Fig. 1). $N_2 + O_2$, 51.7 % O_2, atmospheric pressure. (*b*) V_Q as a function of V ($d = 25$ mm). V_Q and E' obtain maximum values at the same value of V.

between the planes. Owing to this, the shape of the curve 2.0 mm is different from that of the others at low voltages. An increase of the point voltage above 4000 volts causes a further decrease of the critical field strength, but this decrease becomes less and less the higher the voltage. Of course, at a sufficiently high voltage breakdown occurs. What has been said above concerning the dependence of the critical field strength on the point voltage and the point–plane distance is essentially valid also for other gas mixtures and pressures used in this work.

At small distances between the point and the plane the curves are displaced towards lower voltages, but the shape of the curve is essentially the same. Considering this, we can say that a decrease of the point–plane distance causes a decrease of the critical field strength. Thus the maximum value increases from 8.00 kV/cm at a distance of 0.5 mm to 8.75 kV/cm at 2.0 mm. A similar dependence of the critical field strength on the point–plane distance—that is, a change of about 10 % has been observed in other mixtures of N_2 and O_2.

When the measurements of the critical field strength were made in gas mixtures containing argon or helium a somewhat different setup was used than that described above. The reason for this is the following. If the point is placed opposite to the hole, as shown in Fig. 4*a*, it is very difficult to get a self-sustaining streamer discharge in gas mixtures containing argon or helium because breakdown occurs very easily. It was, therefore, necessary to make the diameter of the hole H greater (8 mm) and to place the point as shown in Fig. 4*b*. If this arrangement is used the risk of break-

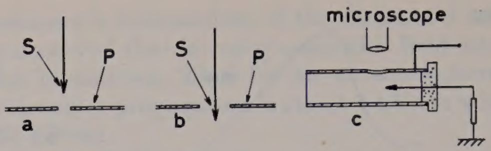

Fig. 4. (*a*) and (*b*) Different arrangements of the point *S* and the plane *P*. The diameter of the hole *H* is (*a*) 3.0 mm, (*b*) 8.0 mm. In (*c*) a tube is used as negative electrode (cf. text).

down is rather small and the critical field strength can be measured. Consequently, this arrangement has been used at the measurements of the critical field strength in the following gas mixtures: $N_2 + A$, $N_2 + H_2 + A$, $N_2 + He$, $A + O_2$, and $A + CCl_4$ (see below).

As we have seen above, the critical field strength is dependent on the point–plane distance. Hence, it is not surprising if the values measured with the point placed as shown in Fig. 4*b* are somewhat different from those obtained with the arrangement in Fig. 4*a*. Experimentally it has been found that this difference is small for low values of the critical field strength but for high values it is more important. Thus in oxygen the measured value of the critical field strength is 14 kV/cm, if the arrangement in Fig. 4*a* is used, and 11 kV/cm, if that in Fig. 4*b* is used. This must be observed when we compare the values of the critical field strength obtained by using the different arrangements in Fig. 4.

3. Measurement of the critical field strength

A preliminary investigation of the possibilities of measuring the critical field strength in gas mixtures showed that the $N_2 + O_2$ mixtures were particularly suitable. Thus streamers are formed if the amount of oxygen exceeds about 35 %, and with the admixture of some per cent of argon streamers are obtained with as low an amount of oxygen as 25 %. The streamer pulses seen on the oscilloscope screen are very regular with the exception of the case where the amount of oxygen exceeds about 95 %. In this case there is a rather large variation in the height of the pulses. The critical field strength has always an easily measurable value, since it is always considerably lower than the field strength necessary for breakdown. There is no difficulty in reproducing the measurements, and the difference between the values of two measurements of the critical field strength is generally less than 3–4 %. Nitrogen and oxygen for use in the measurements were taken from commercial tanks. The gases had been produced by distillation of liquid air.

The arrangement shown in Fig. 1 was used for the measurements, and these were carried out as described on page 445. The diameter of the hole *H* was 3.0 mm and the distance between the point and the plane was 1.5 mm. Measurements have been made in the pressure range 200–760 mm Hg and the amount of oxygen has been varied between 28 % and 100 %. The lowest pressure at which streamers are formed is dependent on the percentage of oxygen but is always greater than about 200 mm Hg. It is higher, the lower the percentage of oxygen. At atmospheric pressure it is hardly possible to obtain streamers at a percentage of oxygen lower than about 35 %. In spite of this it has been possible to make a measurement at 28 %, but in this case an admixture of about 5 % of argon was used. The argon facilitates the

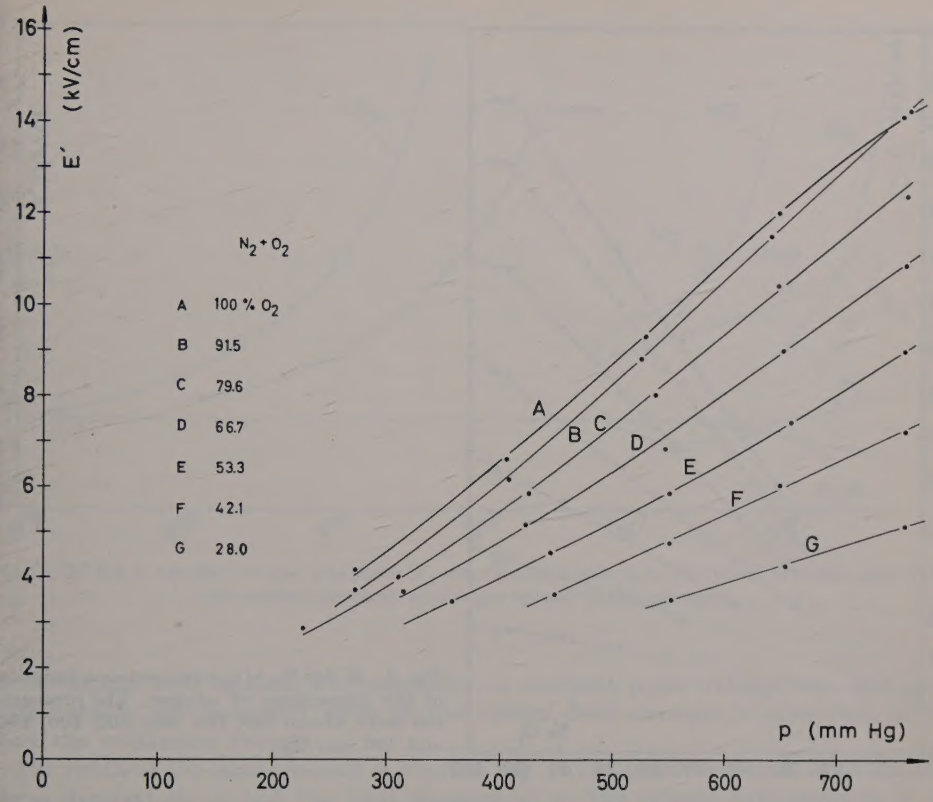

Fig. 5. E' for N_2+O_2 mixtures as a function of the pressure p at different percentages of O_2.

streamer formation but affects the critical field strength only slightly, which has been checked at higher percentages of oxygen.

As the critical field strength is not independent of the point voltage, it is necessary to define a voltage at which the measurements must be made. Otherwise results obtained at different pressures cannot be compared. One possibility is to choose a constant potential independent of the pressure, but this is only possible within a small pressure range, as breakdown occurs at low pressures. Another possibility is to change the point–plane distance and use a constant potential, but this makes it impossible to investigate the whole pressure range within which streamers are formed, as the point voltage at low pressures is too high. In this case no streamers are formed (cf. Part I [2]). Consequently, the voltage must be adjusted to the pressure, and fortunately this can be done by using the relation in Fig. 3.

Even if the voltage range within which streamers are formed is rather small, there is always some value of the point voltage at which the critical field strength has a maximum (on the assumption that the point–plane distance is not too large). Hence the point voltage has been chosen so that the critical field strength is maximum. The simplest way to determine this value of the voltage is to choose a rather large value of d and then adjust the point voltage so that V_Q is maximum. The relation between V_Q and the point voltage is shown in Fig. 3*b* for $d=25$ mm and a

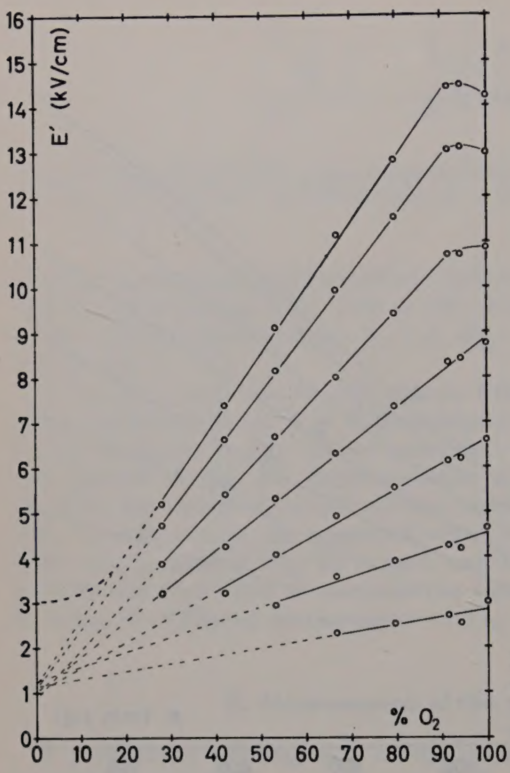

Fig. 6. E' for $N_2 + O_2$ mixtures as a function of the percentage of oxygen. The pressures are from above 760, 700, 600, 500, 400, 300, and 200 mm Hg.

point–plane distance of 1.5 mm. From the figure it is clear that V_Q has its maximum at the same voltage as E'. If d is small, this does not generally hold, because in this case the change of the point voltage affects V_Q to a fairly considerable extent.

The two gases N_2 and O_2 were mixed in the container in the desired proportions, and E' was measured at a number of different pressures. This was then repeated with other mixtures. In Fig. 5 the measured values of E' are plotted against the total pressure of the gas mixture.

From Fig. 5 it is clear that, independent of the percentage of oxygen, there is approximately a linear relation between the critical field strength and the pressure. From the curves in Fig. 5 we can determine the critical field strength at 760, 700, 600, 500, 400, 300, and 200 mm Hg. The values obtained in this way are plotted in Fig. 6, where the critical field strength is shown as a function of the percentage of oxygen at the pressures just mentioned. From the figure it is clear that at constant pressure the critical field strength is a linear function of the percentage of oxygen, if this is less than 92. The straight lines drawn in the figure intersect at a point for which the percentage of oxygen is zero.

The critical field strength was also measured in a number of other gas mixtures (Fig. 7). All these gas mixtures contained nitrogen. As a rule only one other gas was admixtured to the nitrogen. However, when the gas mixture contained hydrogen it was necessary to use an admixture of some per cent of argon, because otherwise

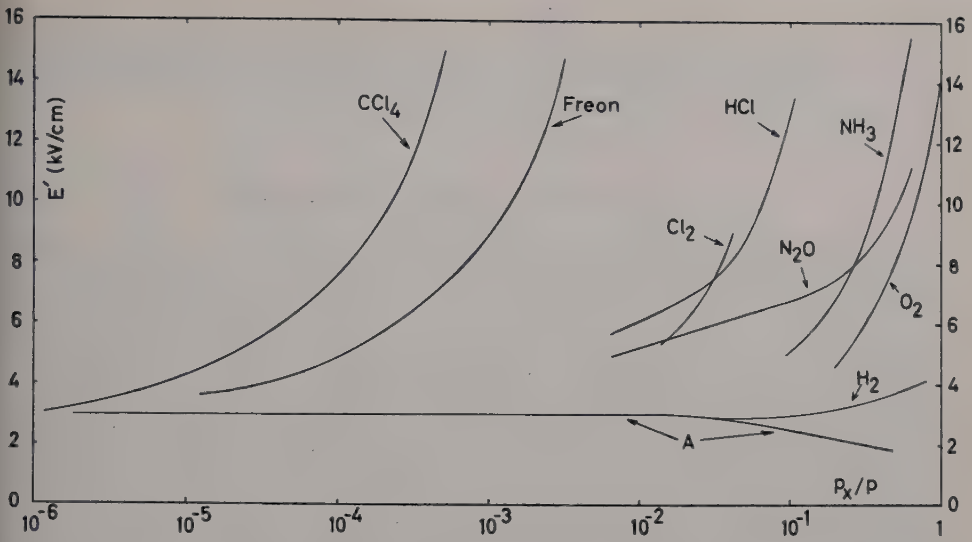

Fig. 7. E' for a number of gas mixtures N_2 plus another gas. p is the total pressure and p_x is the partial pressure of the gas mixed to the nitrogen.

no streamers were formed. For simplicity, a constant point voltage was used and consequently the measured value of the critical field strength is somewhat lower than the maximum value.

The result of the measurement is whon in Fig. 7. The partial pressure of the mixed gas is denoted by p_x and the total pressure by p. The critical field strength E' is given as a function of p_x/p. All the measurements have been made at atmospheric pressure and room temperature. The influence of the different gases on the critical field strength will be discussed below. Here we concentrate our attention on the fact that the admixture of argon and hydrogen has only a relatively small influence on the critical field strength. On the other hand only very small amounts of CCl_4 and freon are needed to increase the critical field strength considerably. The critical field strength in nitrogen + helium mixtures (not shown in the figure) is about the same as that in nitrogen + argon mixtures. The critical field strength in nitrogen with an admixture of small amounts of another gas is 3 kV/cm, independent of which gas is mixed to the nitrogen. No streamers are formed in pure nitrogen.

The rather simple relation (Fig. 6) found above between the critical field strength and the composition of the gas mixture is somewhat surprising. One must take into consideration that in explaining the streamer mechanism it is necessary to resort to many different elementary processes. Thus excitation, photon production, photon absorption, photo-ionization, ionization by electrons, and the formation of positive space charges and negative ions must play an important role in the streamer mechanism. However, the rather simple relation allows us to suppose that only a few or perhaps simply one elementary process plays the determining role for the critical field strength, whereas the effect of the others is more or less unimportant. Unfortunately, our knowledge about most of the elementary processes is still very limited. Consequently, it is impossible to make a quantitative calculation of their influence on the critical field strength. Furthermore, the streamer mechanism is very complicated and only qualitatively explained. We must, therefore, confine ourselves to making a qualitative discussion of the possible influence of the different elementary processes on the critical field strength.

4. What determines the critical field strength?

Before we discuss the influence of the different elementary processes on the critical field strength we must try to understand in what way the applied field affects the field strength in the streamer, and especially the field strength in the head of the streamer. According to the streamer theory [16], we have a very high concentration of positive ions and a comparatively low concentration of electrons in the head of the streamer. This results in a very high field strength in the streamer head and just in front of it. This field strength must be of the order of 100 kV/cm, otherwise the necessary intense ionization would not be possible. However, one can also say that the high field strength gives rise to the low concentration of electrons, and consequently the positive space charge must be great. From this discussion it follows that the high field strength and the great positive space charge are so intimately connected to each other that it is impossible to decide which of them gives rise to the other. Consequently, it is impossible to understand the mechanism of the streamer propagation if we direct our attention solely to the processes in the streamer head. It is necessary to study what happens in the region behind the head of the streamer, because, as will be seen below, the field strength in this region almost completely determines the conditions in the head of the streamer.

Let us suppose that a certain number, n_0, of electrons and an equal number of positive ions are produced per cm of the streamer. The mobility of the positive ions is about 100 times less than that of the electrons [17] and the velocity of the streamer tip is about 10^7 cm/sec (see page 458)—that is, the time during which the ions are affected by a high field strength is very short (of the order of 10^{-9} sec), for the region of high field strength is of the order of 10×10^{-3} cm (Fig. 15). Consequently, the positive ions can be considered to be at rest during the time of formation of the streamer. This means that we have n_0 positive ions per cm of the streamer and a number of electrons that can be expected to be different in different parts of the streamer, for the number of electrons per cm of the streamer, N, varies with the field strength. As will be seen below, N can be greater than n_0 only if the electrons move in the direction from the point to the streamer tip.

At a distance of more than about 1 mm behind the streamer tip, the field strength, owing to the space charge of the streamer, is of the order of the applied field strength (see below). Just behind the head of the streamer the field strength must be rather small because the positive space charge in the head of the streamer gives rise to a field that counteracts the applied field in this region. As mentioned above, the streamer theory is built on the assumption that the field strength is very high in the head of the streamer. Consequently, we can say that the field strength is of the order of the applied field strength in all parts of the streamer with the exception of the streamer head.

If we increase the applied field strength (for instance, from 5 kV/cm to 7 kV/cm) the relative increase of the field strength becomes rather small (some per cent) in the head of the streamer, but in all other parts of the streamer the relative increase becomes much more important. This means that the increase of the drift velocity of the electrons becomes rather important in the whole space behind the streamer head. But if the drift velocity of the electrons behind the streamer head is increased, the number of electrons removed from the streamer head must increase and consequently $n_+ = n_0 - N$ must increase as well—that is the excess positive space charge must increase. Therefore an increase of the applied field strength must affect the field strength in front of and in the streamer head in an indirect way. This indirect action must have a much greater influence on the field strength than the direct addition of the applied field strength. Consequently, the essential task of the applied field must be the removal of electrons from the region just behind the head of the streamer, and it is of great importance that we direct our interest also to the region behind the head of the streamer.

The applied field sufficient for the propagation of the streamer is so weak that the ionization by electrons must be negligible, if no space charge distorts the field. Consequently, almost all the ions must be produced in the head of the streamer and just in front of it. Parallel to this ion production there is also an intense production of photons. According to the streamer theory a great many of these photons are able to ionize the gas. Each of the electrons produced by photo-ionization in front of the streamer starts an avalanche. The size of the avalanches is dependent on the field strength: the higher the field strength, the larger the avalanches and the greater the number of ion pairs produced.

We can now discuss the influence of the different elementary processes on the critical field strength. As mentioned above, the following processes are of importance for the streamer propagation:

(*a*) ionization by photons,
(*b*) ionization by electrons,
(*c*) the drift velocity of the electrons,
(*d*) the formation of negative ions by attachment of electrons to molecules and atoms.

(*a*) *Ionization by photons*

Let us assume that the parameters of all processes with the exception of the photo-ionization are constant. If the photo-ionization increases—that is, if more primary electrons are produced per cm^3 in front of the streamer— the number of avalanches increases. This means that if the field strength is constant the number of ions produced per cm of the streamer increases. This gives rise to an increase in the positive space charge and in the field strength owing to this space charge. Hence it is possible for the streamer to propagate, if the applied field is decreased. In the same way a decrease in the photo-ionization must be compensated by an increase in the applied field strength.

It is very difficult to draw any conclusions concerning the influence of the photo-ionization on the critical field strength, because we do not know whether the photo-ionization increases or decreases, if a certain gas is mixed to the nitrogen. All that we know is the ionization potentials of the different gas molecules. The ionization potentials of the gases of interest here are given in Table 1 [18, 19, 20].

If a gas with low ionization potential is mixed to the nitrogen we would expect that the critical field strength would be lower than if a gas with high ionization potential is mixed to the nitrogen. We would expect that many of the photons emitted

Table 1. Ionization potentials in volts.

He	24.6	H_2O	13.0	CCl_2F_2	11.7
A	15.8	N_2O	12.9	Cl_2	11.5
N_2	15.6	HCl	12.7	CCl_4	11.5
H_2	15.4	O_2	12.2	NH_3	10.3

by the nitrogen molecules would be able to ionize the molecules mixed to the nitrogen if the ionization potential of these is lower than 15.6 volts. These photons are not able to ionize the molecules of a gas, the ionization potential of which is higher than 15.6 volts.

If we study Table 1 we find that only helium and argon have an ionization potential that is higher than that of nitrogen. Consequently, we would expect the critical field strength in $N_2 + He$ and $N_2 + A$ mixtures to be greater than 3 kV/cm. But this is not the case. On the other hand the critical field strength in $N_2 + O_2$, $N_2 + CCl_4$ mixtures and so on is higher than 3 kV/cm, in spite of the ionization potential of O_2, CCl_4, and so on, being lower than 15.6 volts.

The photo-ionization efficiency has been measured by Weissler *et al.* From these measurements [18, page 318], it is difficult to understand why the critical field strength in $N_2 + O_2$ mixtures increases with the amount of oxygen, if it is determined by the photo-ionization. Of course, it is not possible to draw any definite conclusions because we do not know how many photons are generated in the streamer head. The number of photons probably depends on the composition of the gas. But although it is not excluded it seems rather improbable that the variation of the critical field strength is due to a variation of the photo-ionization.

(*b*) *Ionization by electrons*

As several measurements of the first Townsend coefficient α have been made [21–25], it is easier to make conclusions concerning the influence of the ionization by electrons on the critical field strength than to decide whether the photo-ionization determines the critical field strength or not. The value of α is lowest for nitrogen, somewhat higher for air and highest for oxygen. Consequently, if we suppose the field strength to be constant, the ionization must increase with the percentage of oxygen, if α increases. According to the discussion above (page 453), this means that there is no need for an increase in the critical field strength, if the percentage of oxygen is increased. Consequently, it seems very improbable that the variation of α in $N_2 + O_2$ mixtures can give rise to such a great difference in the critical field strength as shown in Fig. 6.

(*c*) *The drift velocity of the electrons*

Let us assume that the parameters of all processes with the exception of the drift velocity of the electrons are constant. The effect of the two processes discussed above is limited to the region of the streamer where the field strength is very high. This is not the case with the processes that will be discussed below. As mentioned above, the excess number of positive charges n_+ is dependent on the drift velocity of the electrons, not only in the head of the streamer but also just behind it. As soon as the electrons have left the space of high field strength their drift velocity is determined to a great degree by the applied field. The more rapidly the electrons move, the smaller is the space charge effected by them and the higher the field strength in the streamer head. The number of electrons per cm of the streamer is derived on page 460. When no negative ions are formed it is

$$N = n_0 \frac{v_t}{v_e'(l) + v_t},$$

where n_0 is the number of ion pairs produced per cm of the streamer, $v_e'(l)$ the drift velocity of the electrons, and v_t the velocity of the streamer tip. Of course, $v_e'(l)$ is a function of the distance to the streamer tip. If the mobility of the electrons is decreased, the drift velocity of the electrons decreases. Consequently, the number of electrons per cm increases and the field strength due to the excess positive space charge decreases. Therefore it is necessary to increase the applied field strength if the streamer is to be able to propagate.

As the mobility of the electrons is different in different gases, its influence on the critical field strength must also be different. The mobility of the electrons is greater in oxygen than in nitrogen [17]. From the values of the mobility of electrons in air we can expect the mobility in a $N_2 + O_2$ mixture to vary approximately linearly with the percentage of oxygen. In order to obtain the same value of $v_e'(l)$, it is necessary to use a higher field at a low percentage of oxygen than at a high one. But this is in contradiction to the experimental facts; and the conclusion must be that this process cannot explain the variation of the critical field strength.

(*d*) *The formation of negative ions*

Now there remains only one process which probably can explain the critical field strength. If electrons in the streamer are attached to molecules and atoms, the negative space charge is increased because the negative ions move slowly compared to the electrons. As the attachment occurs especially behind the head of the streamer, it causes an important lowering of the resulting field strength in this region. This means that the electrons move more slowly than if no negative ions were formed, and the positive space charge is decreased, not only due to the negative ions but also due to an increase of the space charge of electrons. In order to increase the drift velocity of the electrons it is necessary to increase the applied field. Hence, if a gas, the attachment probability of which is high, is mixed to nitrogen, a high critical field strength is to be expected. On the other hand if the attachment probability is low, we must have a low critical field strength. This assumption is supported by the observation that the number of negative ions formed in the streamer is great, if the critical field strength is high. The formation of negative ions in the streamer can be studied on the oscilloscope screen as shown in Part I [2].

Let us now study Fig. 7 and look for some connection between the value of the critical field strength and the ability of the gases mixed to the nitrogen to form negative ions. At first we note that no negative ions are formed in N_2 [26] and the noble gases [18, page 460]. In hydrogen only very few negative ions are formed [27]. From Fig. 7 we see that the critical field strength in $N_2 + A$ mixtures decreases, if the amount of the noble gas increases. The same is the case in $N_2 + He$ mixtures (not shown in the figure). The decrease of the critical field strength with the amount of the noble gas probably depends on the fact that α is much greater in the noble

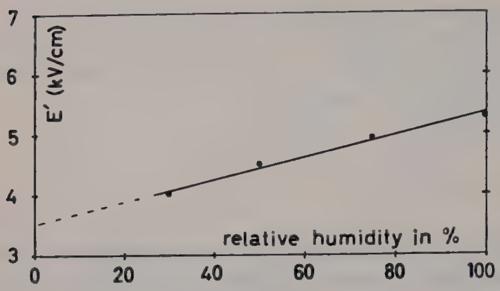

Fig. 8. E' as a function of the relative humidity of the air (room temperature).

gases than in nitrogen [18, pages 530 and 533]. In $N_2 + H_2$ mixtures we find a small increase in the critical field strength with the amount of hydrogen.

When gases that form negative ions are mixed to the nitrogen the critical field strength becomes greater. The attachment coefficient η is about 0.1 attachments/cm at a pressure of 1 mm Hg in oxygen [21, 30], and in ammonia and nitrous oxide it is still greater [28]. This is in good agreement with the assumption that the increase of the critical field strength can be caused by the formation of negative ions in the streamer.

In chlorine and hydrochloric acid gas η is greater than in ammonia and nitrous oxide [28], and we find that also the critical field strength is greater in $N_2 + Cl_2$ and $N_2 + HCl$ mixtures than in $N_2 + NH_3$ and $N_2 + N_2O$ mixtures. η is very great in freon and CCl_4 (greater in CCl_4 than in freon) [28], and also the critical field strength is very high. Negative ions are formed in water vapour [31]. From Fig. 8 we see that the critical field strength increases with the amount of water vapour in the air.

From the discussion above it seems very probable that the negative ions have a great influence on the critical field strength. However, it could be possible that this influence is restricted to the case where the gases that form negative ions are mixed to nitrogen. Therefore it is desirable to study the critical field strength in a gas mixture where the nitrogen is exchanged for another gas that does not form negative ions but in other respects is as different from nitrogen as possible. If we look for such gases we shall find that the noble gases are the most suitable ones. In these gases no negative ions are formed and as they are monatomic these gases behave in other respects quite different from nitrogen. Thus α is much greater in the noble gases than in nitrogen [18, pages 530 and 533].

The measured values of the critical field strength in argon-oxygen mixtures at atmospheric pressure are shown in Fig. 9. The general shape of the curve in Fig. 9 is the same as in Fig. 6. The critical field strength increases in proportion to the percentage of oxygen, if this is lower than about 90 % but the values are less than those in Fig. 6. As regards high values of the percentage of oxygen, the difference is explained by the different arrangements of the point and plane described above (Fig. 4). However, this is not the case when the gas contains small amounts of oxygen, for, as was mentioned before, the different arrangements give rise to only a rather unimportant difference in the critical field strength, if this is low. This is what must be observed when we compare the critical field strength in $A + O_2$ mixtures (Fig. 9) with that in $N_2 + O_2$ mixtures (Fig. 6).

A measurement of the critical field strength in an $A + CCl_4$ mixture was also made,

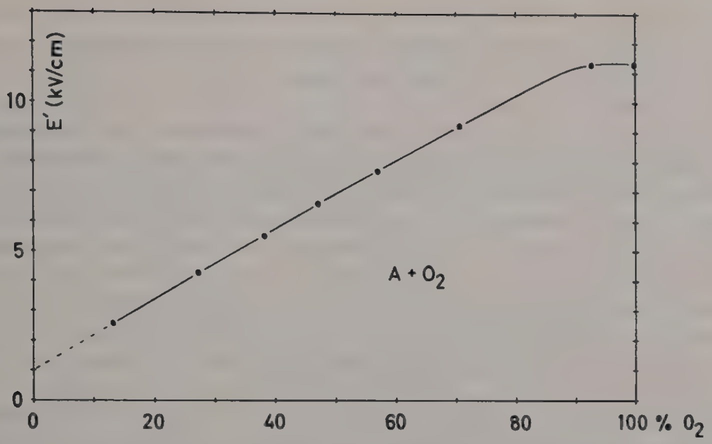

Fig. 9. E' in $A + O_2$ mixtures as a function of the percentage of oxygen (atmospheric pressure).

and E' was found to be about the same as in a $N_2 + CCl_4$ mixture with the same percentage of CCl_4.

Consequently, it can be concluded that the influence of the oxygen on the critical field strength is the same whether it is mixed to nitrogen or argon. In the same way we find that the influence of the argon is about the same if it is mixed to nitrogen as if it is mixed to oxygen. Furthermore we can interprete the influence of the nitrogen as independent of the gas that is mixed to it (Fig. 7). This is in very good agreement with the hypothesis that the formation of negative ions in the gas must have a considerable influence on the critical field strength.

From this study we see that the critical field strength and the electron attachment coefficient increase in the same order. Of course, this may be an accidental occurrence but if we try to calculate the probability of this we will find that this probability is rather small. According to the values of η we can divide the nine gas mixtures into six groups: (1) $N_2 + A$ and $N_2 + H_2$ mixtures, (2) $N_2 + O_2$ mixtures, (3) $N_2 + NH_3$ and $N_2 + N_2O$ mixtures, (4) $N_2 + Cl_2$ and $N_2 + HCl$ mixtures, (5) N_2 + freon mixtures, and (6) $N_2 + CCl_4$ mixtures. Let us suppose that there is no connection between the critical field strength and the ability of the gases to form negative ions. If there was only one gas mixture in each group the probability that the critical field strength will increase in the same order as η would be $1/6! = 1/720$. But in this case we have two gas mixtures in three of the groups and therefore the probability is smaller than 1/720. As the gases have been chosen only according to the principle that they should not be too difficult to obtain, the calculation must be valid. Consequently, the probability of an accidental occurrence is very small.

Now it would be of especial interest to know if the formation of negative ions can cause a field strength in the streamer that is of the order of the critical field strength. Of course it would be most satisfactory to make an experimental determination of the field strength owing to the negative ions but this surely is impossible. Hence, we must try to make a theoretical estimation, but before we can do this we must determine the charge produced per cm of the streamer and the velocity of the streamer tip.

5. The velocity of the streamer tip and the charge produced per cm of the streamer

In the calculations below we need numerical values of $v_t/v_e(l)$, where v_t is the velocity of the streamer tip and $v_e(l)$ the drift velocity of the electrons. However, an experimental determination of $v_e(l)$ is impossible and we must confine ourselves by measuring v_t and estimating $v_e(l)$ (see below, page 468). v_t was measured and was found to be 1.3×10^7 cm/sec. The measurement was carried out by using a photomultiplier in about the same way as described by Amin [12]. However, it was possible to expose the photomultiplier to the whole gap between the planes P and Q (Fig. 1). When the streamer reached the plane Q, there was a sudden increase in the photon pulse. The position of this on the oscilloscope screen was plotted against the distance between the planes P and Q (Fig. 1), and v_t was determined from the slope of the line.

The measurement of the charge produced per cm of the streamer is rather difficult because the electrons that are produced in the streamer rapidly reach the space in front of the point where they start avalanches. This means that the current is increased. Since in addition to the current from the streamers we have a current from the continuous discharge, a measurement of the charge of the streamer can only be made by measuring the voltage pulse due to the electrons. Moreover, it must be observed that if the length of the streamer is more than one or two millimetres, many of the electrons produced furthest from the point form negative ions and do not contribute to the electron pulse. Furthermore, as the drift velocity of the positive ions is about 100 times less than that of the electrons, the positive space charge formed in front of the point affects electrostatically the potential of the point.

As the length of the streamers varies rather greatly with the composition of the gas, the arrangement shown in Fig. 4*c* was used for the measurement. The point was grounded via a resistance of 1035 Ω and the cylindrical electrode was used as the cathode. The potential variation of the point was studied with a Tektronix 541 oscilloscope with a sweep speed of 10 cm/μsec and a rise time of 10^{-8} sec. The charge of the streamer was determined by integration of the voltage pulse over the time of duration. Measurements made with streamers of less than 6 mm in length have given 4–5×10^{-10} coulombs/cm at 60 % $N_2 + 40$ % O_2 and the same value has been obtained at 100 % O_2 (760 mm Hg). This means that about 3×10^9 electrons are produced per cm of the streamer. This value is lower than that reported by Kip [4] and Amin [12] (6–8×10^9 electrons/cm) for pre-onset streamers, which is certainly due to the different shaping of the point and the smaller dimensions that are used here. This is in good agreement with Amin's measurements, according to which the charge per cm is less, the smaller the diameter of the point. Measurements have also been made in other $N_2 + O_2$ mixtures, and about the same value of the charge per cm as given above has been obtained. Of course, as mentioned above, the measured values can only give an approximate value of the charge produced per cm of the streamer.

It is possible to determine the upper limit of the charge produced per cm of the streamer. The charge produced per second in the streamers must, of course, be smaller than the total current of the discharge. By measuring the total current, the number of the streamers formed per second, and the length of the streamers, it was found that the charge produced in the streamers is less than 8×10^{-10} coulombs/cm.

As in the following calculations the streamers are assumed to propagate in a homogeneous field it would be desirable to know the charge produced per cm of the streamer in this case. A measurement of the number of streamers and the current to the plane Q (Fig. 1) does not give a correct value of the charge per cm of the streamer, because each time a streamer reaches the plane Q a number of electrons are emitted by it. In spite of this, such a measurement was performed and a correction was made for the charge emitted by the plane Q. In this way it was found that the charge produced per cm of the streamer must be less than 8×10^{-10} coulombs/cm also in this case.

B. THEORETICAL CALCULATIONS

Outline of calculations

As is clear from the experimental part, the critical field strength increases in the same order as the formation of negative ions in the gas. Therefore, it is to be expected that the increase in the critical field strength is due to the formation of negative ions. However, a necessary condition for this is that the field strength due to the negative ions formed in the streamer should be of the same order as the critical field strength. Below, two different calculations will be carried out. The purpose of these is to give information about the order of magnitude of the field strength due to the formation of negative ions in the streamer.

In the first calculation (*a*), we start with calculating the number of electrons per cm in the streamer as a function of their drift velocity. This is made (I) when no negative ions are formed, Eq. (2), and (II) when negative ions are formed, Eq. (5). In the latter case also the number of negative ions formed per cm in the streamer is calculated and expressed as a function of the drift velocity of the electrons $v_e(l)$, the attachment coefficient η and the distance to the streamer tip l, Eq. (4). It is then possible to obtain an expression for the total number of negative charges per cm by adding the number of electrons and the number of negative ions, Eq. (6). The increase in the number of negative charges per cm due to the formation of negative ions is now obtained as the difference between the total number of negative charges per cm when negative ions are formed and the number of electrons per cm when no negative ions are formed, Eq. (7).

The field strength due to the increase of the number of negative charges must be compensated by an increase in the applied field, if the field strength in the streamer is to be the same when negative ions are formed as when no negative ions are formed. Consequently, we want to calculate the field strength due to the increase in the negative space charge. Of course, it would be desirable to perform this for the general case—that is, without making any assumptions concerning the dependence of $v_e(l)$ on the distance to the streamer tip. However, this is impossible and in the following calculations only the most simple case is treated—that is, when $v_e(l)$ is a constant independent of the distance to the streamer tip.

In section 2 the field strength due to a cylindrical space charge is derived, Eq. (13), and in section 3 the result of this calculation is used to derive an expression for the field strength due to the increase in the negative space charge when negative ions are formed, Eq. (15). However, this expression contains v_e and we must determine what values of v_e are probable in the streamer. The drift velocity of the electrons is determined by the applied field and the field due to the space charges in the streamer. In section 5 the field strength due to the excess positive space charge is calculated and an expression for v_e is derived. This is used in section 6 to calculate the field strength due to the increase of the negative space charge. This field strength is obtained as a function of the charge produced per cm in the streamer. It is calculated for a number of different values between zero and infinity of the field strength due to the excess positive space charge in the streamer, Fig. 16. From these calculations it can be concluded that the field strength due to the increase in the negative space charge must be of the same order of magnitude as the critical field strength.

In section (*b*) another calculation is carried out. In this case we start by asking: How much must we increase the applied field strength if we want to maintain the same number of negative charges per cm of the streamer if negative ions are formed as if no negative ions are formed? If the negative space charge is to be the same in both cases the drift velocity of the electrons must be greater when negative ions are formed than when no negative ions are formed. The difference between the drift velocity of the electrons in the two cases can be expressed as a function of $v_e'(l)$, η, and l. In this case $v_e'(l)$ is considered as an arbitrary function of l. As the drift velocity of the electrons is proportional to the field strength [17], the necessary increase E_3 in the field strength can also be expressed as a function of $v_e'(l)$, η, and l, Eq. (20). Since by definition the field strength due to the space charges in the streamer is the same when negative ions are formed as when no negative ions are formed the expression for E_3 is of interest only if E_3 is small. However, E_3 is small only if η is small and consequently it is better to study $(dE_3/d\eta)_{\eta=0}$ than E_3. In the expression for $(dE_3/d\eta)_{\eta=0}$, $v_e'(l)$ is substituted by suitable functions of l and numerical values of $(dE_3/d\eta)_{\eta=0}$ are calculated. These are then compared with the experimentally determined values for $N_2 + O_2$ and $A + O_2$ mixtures.

(*a*) The field strength due to the increase of the negative space charge when negative ions are formed

1. *The number of negative ions formed in the streamer*

The number of negative charges per cm in the streamer will now be determined as a function of the drift velocity of the electrons. Let us assume that the streamer tip advances from left to right in Fig. 10 with constant velocity v_t. Let us further assume that the velocity of the electrons $v_e(x)$ is only a function of the distance to the streamer tip x. The field strength varies with the distance to the streamer tip and therefore $v_e(x)$ varies as well. Suppose that the streamer tip has reached A at time $t = 0$, A' at time $t = dt$ and B at time $t = T$. If we direct our interest to the electrons that have not been attached to gas molecules or atoms we have: The electrons that were produced at A have reached C at time $t = T$. The electrons that at time $t = dt$ were produced at A' are at time $t = T$ at C'. Evidently the electrons that were produced between A and A' have passed C' but not C. The electrons that were produced at A' have moved during the time $T - dt$.

Hence the distance $$C'A' = \int_0^{T-dt} v_e(x) \cdot dt.$$

In the same way the distance

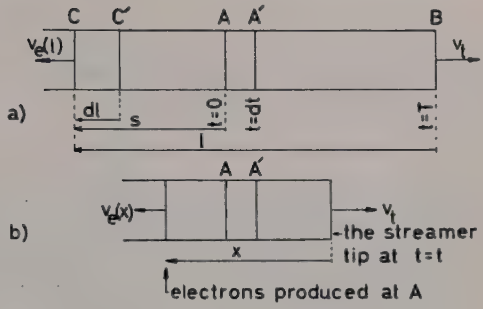

Fig. 10. Figure for the derivation of Eqs. (2) and (6).

$$C\,A = \int_0^T v_e(x)\cdot dt.$$

It is to be observed that x is a function of t and consequently $v_e(x)$ is a function of t.

The distance $AA' = v_t \cdot dt$. But

$$\begin{aligned} C\,C' &= C\,A + A\,A' - C'\,A' \\ &= \int_0^T v_e(x)\cdot dt + v_t\cdot dt - \int_0^{T-dt} v_e(x)\cdot dt \\ &= v_e(l)\cdot dt + v_t\cdot dt. \qquad (1) \end{aligned}$$

If n_0 electrons are produced per cm in the streamer, the number of electrons produced between A and A' is $n_0 \cdot v_t \cdot dt$. If the number of electrons per cm at C is N, if no negative ions are formed, we obtain

$$N = n_0 \cdot \frac{v_t}{v_e'(l) + v_t}, \qquad (2)$$

where $v_e'(l)$ is the drift velocity of the electrons at C if no negative ions are formed.

From Eq. (1) and Fig. 10 we get $CC' = dl = (v_e(l) + v_t)\cdot dt$. Consequently,

$$dt = \frac{dl}{v_e(l) + v_t} \quad \text{and} \quad T = \int_0^l \frac{dl}{v_e(l) + v_t},$$

where T is the time during which the streamer tip has advanced from A to B and the electrons have moved from A to C. The distance CB is l. The distance CA can be written $(l - v_t \cdot T)$ and consequently

$$C\,A = l - v_t \cdot \int_0^l \frac{dl}{v_e(l) + v_t} = \int_0^l \left(1 - \frac{v_t}{v_e(l) + v_t}\right) dl = \int_0^l \frac{v_e(l)}{v_e(l) + v_t}\, dl = s. \qquad (3)$$

At time $t = T$ the electrons that were produced between A and C have either passed C or formed negative ions. Therefore we can use Eq. (5) in Part I [2] when we calculate the number of negative ions, n_-, per cm of the streamer. This equation is

$$n_- = n_0 (1 - e^{-\eta \cdot l}).$$

However, instead of the whole distance l to the streamer tip we must use the reduced distance CA. Therefore we get

$$n_- = n_0 (1 - e^{-\eta \cdot s}) \tag{4}$$

Here η has been considered as a constant independent of the distance to the streamer tip. As η generally is dependent on the value of E/p, this is an approximation but it is necessary because otherwise the calculation would be too complicated.

The total number of negative charges per cm of the streamer is the sum of the negative ions and the electrons. If no negative ions were formed, the number of electrons between C and C' would be $n_0 \cdot AA' = n_0 \cdot v_t \cdot dt$; but as a consequence of the formation of negative ions this is reduced to

$$n_0 \cdot v_t \cdot dt \cdot e^{-\eta \cdot s}.$$

The number of electrons per cm at C is therefore

$$n_e = \frac{n_0 \cdot v_t \cdot dt \cdot e^{-\eta \cdot s}}{CC'} = n_0 \frac{v_t}{v_e(l) + v_t} e^{-\eta \cdot s}, \tag{5}$$

and the total number of negative charges

$$N(\eta) = n_- + n_e = n_0 \left(1 - \frac{v_e(l)}{v_e(l) + v_t} e^{-\eta \cdot s}\right). \tag{6}$$

If no negative ions are formed in the streamer, the number of negative charges per cm of the streamer is equal to the number of electrons per cm. This is given in Eq. (2) but can, of course, also be obtained by substituting $\eta = 0$ in Eq. (6).

Let us now suppose that we have two streamers that propagate in two different gases that are exactly alike with the exception that negative ions are formed in one of them, $(\eta_1 = \eta)$, but not in the other, $(\eta_2 = 0)$. Let us denote the drift velocity of the electrons in the gas where no negative ions are formed by $v_e'(l)$ (at a distance l from the streamer tip) and in the gas where negative ions are formed by $v_e(l)$. We suppose that all other parameters are the same in both cases. The difference between the number of negative charges per cm in the streamer, if negative ions are formed and if no negative ions are formed is

$$N(\eta) - N(\eta = 0) = n_0 \left(\frac{v_e'(l)}{v_e'(l) + v_t} - \frac{v_e(l)}{v_e(l) + v_t} e^{-\eta \cdot s}\right). \tag{7}$$

If $v_e'(l) = v_e(l)$—that is, if the field strength is the same in both cases—the difference is

$$N(\eta) - N(\eta = 0) = n_0 \cdot \frac{v_e(l)}{v_e(l) + v_t} \cdot \left(1 - e^{-\eta \cdot s}\right). \tag{8}$$

This means that the number of negative charges per cm is greater if negative ions are formed than if no negative ions are formed. But the increase of the negative

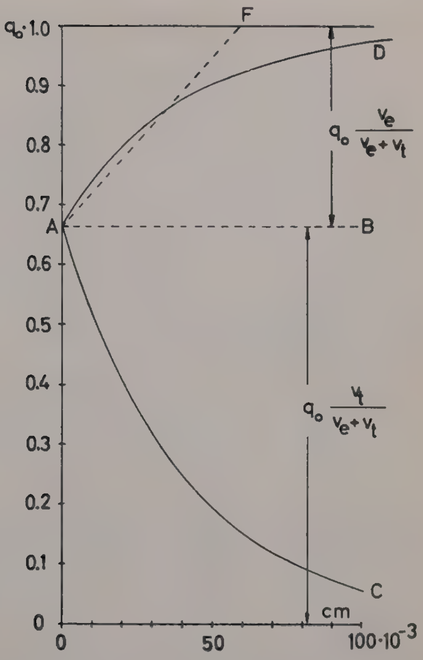

Fig. 11. The charge distribution in the streamer as a function of the distance to the streamer tip. $v_t/v_e = 2.0$, $q_0 = n_0 \cdot e$.

space charge gives rise to a field strength that has a negative value. The field strength is counted as positive if it has the same direction as the applied field strength. We have supposed that the field strength is the same in both cases. Consequently, the field strength owing to the increase of the negative space charge must be compensated by an increase of the applied field strength. Otherwise the field strength in the streamer cannot be the same in the two cases.

As mentioned before, the drift velocity of the electrons must vary with the distance to the streamer tip (l) and it is very difficult to calculate $v_e(l)$ as a function of l. Therefore, if we want to illustrate the influence of the formation of negative ions on the space charge distribution, we must choose $v_e(l)$ as some function of l. Let us start with the most simple case and assume that $v_e(l)$ is constant, independent of l —that is $v_e(l) = v_e$. Of course, this is a very crude approximation but if we choose a reasonable value of v_e we can get a rather good understanding of the influence of the negative ions.

If these assumptions are made the distribution of electrons and negative ions is illustrated in Fig. 11 ($\eta = 76$ cm^{-1}, which is the value for oxygen at atmospheric pressure, $v_t/v_e = 2.0$). At $l = 0$ there are no negative ions and the electron density is $n_0[v_t/(v_e + v_t)]$ electrons per cm. If no negative ions are formed—that is, if $\eta = 0$— the electron density is constant and equal to the value just mentioned, independent of the value of l. If negative ions are formed the electron density decreases with l but the density of negative ions increases so rapidly that the density of negative charges increases according to the curve *A–D*.

We are most interested in the increase of the number of negative charges owing to the formation of negative ions. This increase is given by Eq. (8) and consists of the part that lies above the line *A–B* (Fig. 11). The field, owing to this space charge, counteracts the movement of the electrons and must be balanced by an increase in the applied field, if the electrons are to have the same drift velocity as if no negative ions were formed.

In reality the drift velocity of the electrons is not constant. It is greatest in the head of the streamer and it is comparatively low in the region just behind the streamer head. In the foremost part of the streamer head the field strength is so high that no negative oxygen ions can be formed, for if E/p is greater than about 90 volts/cm mm Hg, these ions are split into electrons and neutral particles [29]. In the centre of the streamer head and in the region behind it the field strength is lower and negative ions can be formed. As in this part of the streamer the field strength decreases with the distance to the streamer tip, $v_e(l)$ decreases as well. Consequently, the space charge distribution differs from that shown in Fig. 11. This difference will now be discussed.

From Eq. (8) it is clear that $N(\eta) - N(\eta = 0)$ is positive for all positive values of $v_e(l)$. Eq. (8) does not hold for negative values of $v_e(l)$ but such values are improbable in this case. $N(\eta) - N(\eta = 0)$ is equal to zero, only if $v_e(l)$ equals zero. If l is constant (s is a function of l; see Eq. (3)), $N(\eta) - N(\eta = 0)$ increases with increasing values of $v_e(l)$. As the value of $v_e(l)$ in the centre of the streamer head must be much greater than the value used in Fig. 11 (this value of v_e is obtained if the field strength is about 7 kV/cm, atmospheric pressure), $N(\eta) - N(\eta = 0)$ must increase more rapidly with l than in this figure. This means that the increase of negative space charge owing to the formation of negative ions is rather important in the back part of the streamer head. In the region just behind the streamer head, $v_e(l)$ is probably somewhat smaller than the value used in Fig. 11. However, $N(\eta) - N(\eta = 0)$ decreases more slowly than $v_e(l)/[v_e(l) + v_t]$ for the high values of $v_e(l)$ at small distances to the streamer tip increase the factor $(1 - e^{-\eta \cdot s})$ at great distances to the streamer tip. Consequently, the increase of the negative space charge owing to the formation of negative ions in the streamer must be of the order of that shown in Fig. 11. Of course, this makes the removal of the electrons from the streamer head more difficult than if no negative ions are formed.

2. *The field strength owing to a cylindrical space charge*

In order to get a quantitative estimation of the field strength in the streamer, the field strength due to a cylindrical space charge will be derived. For simplicity the space charge density will be considered as constant in radial direction in the streamer. Furthermore, the field strength will be calculated only for points on the axis where the radial component equals zero.

Let O–Q be the axis of the cylindrical space charge A, B, D, C (Fig. 12). It is assumed that the charge density q (charge per unit length of the streamer) is constant between A and B. From B to C it decreases linearly to zero. The distances are $AB = L$, $BD = DC = b/2$. Let us assume a ring-shaped space element with the radius r, the breadth dr, and the length dx. The field strength is calculated for the point P at a distance a from D. The distance from P to the centre of the space element is x. The charge dq of the space element contributes to the field strength with

$$dE = \frac{1}{4\pi\varkappa_0} \cdot \frac{dq}{d^2} \cdot \cos\vartheta, \tag{9}$$

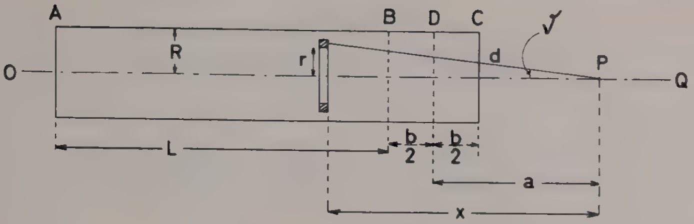

Fig. 12. Figure for the derivation of Eq. 13.

where d is the distance between P and the space element, and ϑ is the angle between the axis O–Q and the direction to the space element. But we have

$$dq = q\frac{2r\,dr}{R^2}\,dx, \qquad d=\sqrt{r^2+x^2}, \qquad \cos\vartheta = \frac{x}{d},$$

where q is the charge per unit of length (q is a function of x), and R is the radius of the cylinder. The expression (9) can be written

$$dE = \frac{1}{4\pi\varkappa_0}\cdot q\cdot\frac{2r\,dr}{R^2}\cdot\frac{x}{(x^2+r^2)^{\frac{3}{2}}}\,dx,$$

and the field strength is

$$E=\frac{1}{2\pi\varkappa_0}\int\limits_{x=a-\frac{1}{2}b}^{x=L+a+\frac{1}{2}b}\int\limits_{r=0}^{r=R} q\,\frac{r\cdot x\,dr\,dx}{R^2\,(x^2+r^2)^{\frac{3}{2}}}. \tag{10}$$

After intergration of (10), we have

$$E=\frac{1}{2\pi\varkappa_0 R^2}\int\limits_{x=a-\frac{1}{2}b}^{x=L+a+\frac{1}{2}b} q\left(1-\frac{x}{\sqrt{R^2+x^2}}\right)dx, \tag{11}$$

which gives the field strength if $a \geqslant \frac{1}{2}b$—that is, if P is situated outside the space-charge. The expression within the parenthesis is correct only for positive values of x.

The value of q is

$$q=q_1 \quad \text{if } x \geqslant a+\tfrac{1}{2}b,$$

$$q=q_1\cdot\frac{1}{b}\,[x-(a-\tfrac{1}{2}b)] \quad \text{if } a-\tfrac{1}{2}b \leqslant x \leqslant a+\tfrac{1}{2}b.$$

If these values of q are used in (11), we obtain

$$E_1 = \frac{1}{2\pi\varkappa_0 R^2}\left(q_1 \int\limits_{x=a+\frac{1}{2}b}^{x=L+a+\frac{1}{2}b} \left[1-\frac{x}{\sqrt{R^2+x^2}}\right] dx + \right.$$

$$\left. + q_1 \frac{1}{b} \int\limits_{x=a-\frac{1}{2}b}^{x=a+\frac{1}{2}b} \left[x-(a-\tfrac{1}{2}b)\right]\left[1-\frac{x}{\sqrt{R^2+x^2}}\right] dx\right)$$

When this expression is integrated we find that the influence of L on the value of E_1 is unimportant, if $L \gg a$ and R. For simplicity let us assume that L is infinite. Then we have

$$E_1 = \frac{q_1}{4b\pi\varkappa_0}\left[\frac{a+\frac{1}{2}b}{\sqrt{R^2+(a+\frac{1}{2}b)^2}+(a+\frac{1}{2}b)} - \frac{a-\frac{1}{2}b}{\sqrt{R^2+(a-\frac{1}{2}b)^2}+(a-\frac{1}{2}b)} + \right.$$

$$\left. + \ln\frac{\sqrt{R^2+(a+\frac{1}{2}b)^2}+(a+\frac{1}{2}b)}{\sqrt{R^2+(a-\frac{1}{2}b)^2}+(a-\frac{1}{2}b)}\right], \qquad (12)$$

which holds if

$$a \geqslant \tfrac{1}{2}b \quad \text{and } L \gg a \text{ and } R.$$

A derivation of the field strength for other points on the axis OQ shows that E_1 is symmetric with respect to point D on the axis. This is the reason for the choice of a as the distance DP and not CP. We find that the field strength can be written

$$E_1 = \frac{q_1}{4b\pi\varkappa_0}\left[\frac{(a+\frac{1}{2}b)}{\sqrt{R^2+(a+\frac{1}{2}b)^2}+(a+\frac{1}{2}b)} - \frac{(a-\frac{1}{2}b)}{\sqrt{R^2+(a-\frac{1}{2}b)^2}+|a-\frac{1}{2}b|} + \right.$$

$$\left. + \ln\frac{\sqrt{R^2+(a+\frac{1}{2}b)^2}+(a+\frac{1}{2}b)}{\sqrt{R^2+(a-\frac{1}{2}b)^2}+(a-\frac{1}{2}b)}\right], \qquad (13)$$

where $|a-\frac{1}{2}b|$ is the absolute value of $(a-\frac{1}{2}b)$. The equation holds for $a \geqslant 0$, and therefore a is considered as positive in both directions.

3. *The change in the field strength owing to the formation of negative ions*

We shall now calculate the change in the field strength owing to the formation of negative ions, if $v_e(l)$ is considered as a constant independent of the distance to the streamer tip. The increase in the negative space charge owing to the formation of negative ions is given by Eq. (8). The change in the field strength owing to this change in the space charge is obtained, if the value of q corresponding to Eq. (8) is substituted in Eq. (11). However, in this case one obtains an equation of the type $\int[1-x/(R^2+x^2)^{\frac{1}{2}}]e^{kx}dx$, which cannot be solved exactly. For simplicity, an approximation will be made as follows. Let us assume that the space charge does not vary exponentially as in Eq. (8) with the distance to the streamer tip but according to

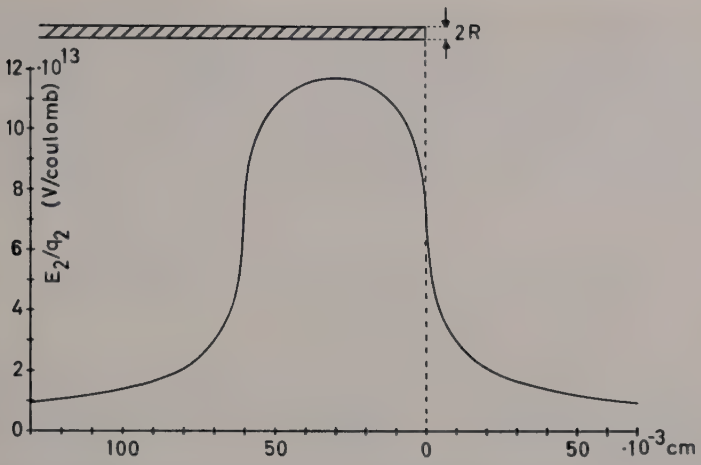

Fig. 13. The field strength due to the increase of the negative space charge, if negative ions are formed (cf. text). At the top of the figure the streamer is shown in correct proportions.

$$q = q_0 \cdot \frac{\eta}{1.5} \cdot \left(\frac{v_e}{v_e + v_t} \right)^2 \cdot l, \quad \text{if } 0 \leqslant l \leqslant 1.5 \, \frac{v_e + v_t}{\eta \cdot v_e} \tag{14a}$$

$$q = q_0 \, \frac{v_e}{v_e + v_t} = q_2, \quad \text{if } l \geqslant 1.5 \cdot \frac{v_e + v_t}{\eta \cdot v_e}, \tag{14b}$$

where q_0 is the total charge produced per unit length of the streamer. This means that the charge rises as the straight line AF in Fig. 11 instead of as the curve AD.

The value of the field strength E_2 is obtained by substituting $b = 1.5[(v_e + v_t)/\eta v_e]$ and $q_1 = q_2$ in Eq. (13). As η increases with the partial pressure of oxygen, the smallest value of b in a $N_2 + O_2$ mixture is obtained at 100% O_2. According to [21, 30] $\eta = p(O_2) \cdot 0.10$ cm^{-1} at the E/p values that can be expected to occur in the streamer (about 10 volts/cm mm Hg). If we assume that $v_t/v_e = 2.0$ and $p(O_2) = 760$ mm, we have $b = 60 \times 10^{-3}$ cm. This is the lowest value of b to be expected in $O_2 + N_2$ mixtures. As v_t is somewhat greater than 10^7 cm/sec (see page 458), $v_t/v_e = 2.0$ corresponds to a field strength of about 7 kV/cm, for $v_e = k \cdot E$, where k is the mobility [17].

Fig. 13 shows the calculated value of E_2/q_2 for $b = 60 \times 10^{-3}$ cm and $R = 2 \times 10^{-3}$ cm. This value of R has been chosen because according to Fig. 3, Part I, the diameter of the streamer cannot exceed 10×10^{-3} cm, but is probably much smaller. It is rather improbable that the diameter is less than 1×10^{-3} cm for in this case the radial field strength would be of the order of 100 kV/cm. If q_2 equals 10^{-10} coulombs/cm, E_2 is of the order of 10 kV/cm, which is of the same order as the critical field strength. As mentioned above, the charge produced per cm of the streamer, q_0, is about 4×10^{-10} coulombs/cm, which gives $q_2 = 1.3 \times 10^{-10}$ coulombs/cm if $v_t/v_e = 2.0$.

In order to make clear that the field strength is appreciable in a "large" part of the streamer, the "streamer" is shown in correct proportion at the top of Fig. 13. The length of the high field region equals b and must be of great importance for the drift velocity of the electrons. If no negative ions are formed, this field strength is absent. As a comparison it may be mentioned that the essential ionization in the streamer head must occur in a region smaller than 10×10^{-3} cm (see below, Fig. 15).

As the maximum in Fig. 13 is rather broad, it is sufficient to determine the maximum value when we want to study the field strength as a function of η, i.e., we can substitute $a = 0$ in Eq. (13). Furthermore, the values of b that will be used are $\geqslant 60 \times 10^{-3}$ cm, i.e. $b \gg R$. Hence the equation can in this case be approximated to

$$\frac{E_2}{q_0} = \frac{1}{4\pi\varkappa_0} \, \frac{\eta}{1.5\left(1+\frac{v_t}{v_e}\right)^2} \left[1 + 2\ln \frac{1.5\left(1+\frac{v_t}{v_e}\right)}{R\eta}\right]. \tag{15}$$

q_1 and b of Eq. (13) have been substituted by $q_0[v_e/(v_e+v_t)]$ according to (14b) and $1.5[(v_e+v_t)/\eta v_e]$, respectively. As is clear from Eq. (15) E_2/q_0 is greatly dependent on the value of v_t/v_e.

4. *Determination of v_t/v_e*

As described above the velocity of the streamer tip v_t was experimentally determined and its value is 1.3×10^7 cm/sec. The value of v_e cannot be measured directly, but we can determine it if we know the field strength and the mobility of the electrons. The field strength in the streamer is the sum of the applied field and the field due to the space charges in the streamer. The space charges can be considered to be a positive space charge with the charge $q_0[v_e/(v_e+v_t)]$ per unit length (cf. Fig. 11) and a negative space charge according to Eq. (14a, b). ($q_0[v_e/(v_e+v_t)]$ is the excess positive space charge if no negative ions are formed). Of course, besides these there are another negative and another positive space charge (see Fig. 11, the charge below the dotted line AB), but these balance each other and can be excluded as long as the field strength is concerned.

The field strength due to the positive space charge can be written

$$E_1 = X \cdot q_0 \cdot \frac{v_e}{v_e+v_t},$$

where X is a constant that is dependent on the distance to the streamer tip. The value of X is determined by Eq. (13), $X = E_1/q_1$. According to [17] the drift velocity of the electrons is approximately proportional to E/p and consequently we can write

$$v_e = k(E' - E_2 + E_1),$$

where k is the mobility, E' the applied field strength, E_2 the field strength owing to the negative space charge, and E_1 the field strength owing to the positive space charge. Hence

$$v_e = k\left[E' - E_2 + X \cdot q_0 \cdot \frac{v_e}{v_e+v_t}\right].$$

If this equation is solved for v_e, we obtain

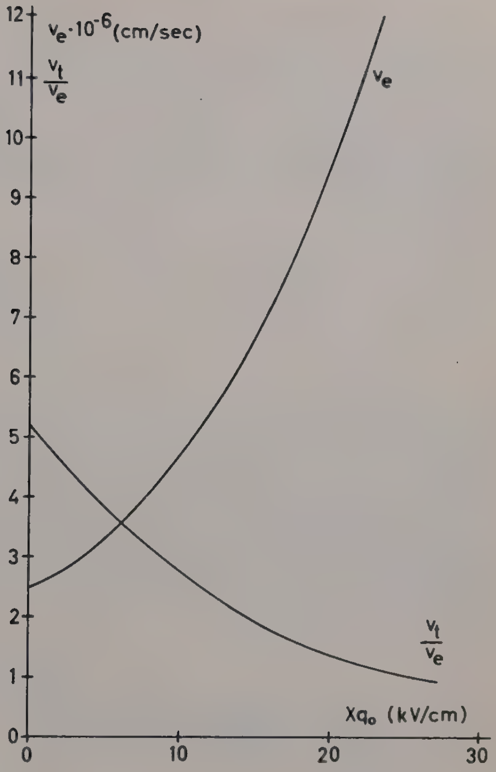

Fig. 14. The drift velocity of the electrons as a function of $X \cdot q_0$ (cf. text).

$$v_e = \frac{k(E' - E_2) + kXq_0 - v_t}{2} + \sqrt{\frac{[k(E' - E_2) + kXq_0 - v_t]^2}{4} + k(E' - E_2)v_t}. \qquad (16)$$

As v_t is known, v_t/v_e can be determined as a function of Xq_0. But first we must choose a value of $(E' - E_2)$. As we have assumed the field strength due to the increase in the negative space charge to be compensated by an increase in the applied field strength, the value of $(E' - E_2)$ is the same if negative ions are formed as if no negative ions are formed. Consequently, we can calculate v_e for the case when no negative ions are formed. In this case E_2 equals zero and $(E' - E_2)$ is equal to the measured critical field strength. This is the case in a mixture of nitrogen and a few per cent of argon but also in mixtures of nitrogen and very small amounts of CCl_4 or freon. As is clear from Fig. 7, the critical field strength is 3.0 kV/cm in these mixtures. As this value is reached asymptotically from both above and below, it is reasonable to conclude that if streamers were formed in a mixture of nitrogen and some per cent of oxygen, the critical field strength would be 3.0 kV/cm at a pressure of 760 mm Hg.

The values of v_e and v_t/v_e, calculated from Eq. (16), are shown in Fig. 14. The values employed in the calculation are $v_t = 1.3 \times 10^7$ cm/sec, $(E' - E_2) = 3.0$ kV/cm, and $k = 8.3 \times 10^2$ cm²/volt·sec, which is the mean value of k for nitrogen and oxygen at atmospheric pressure [17]. As is clear from Fig. 14, v_e increases rather rapidly with Xq_0, and v_t/v_e decreases correspondingly.

According to Eq. (16), v_t/v_e is a function of Xq_0, and according to Eq. (15), E_2 is a function of q_0 and v_t/v_e. Hence E_2 is a function of X and q_0. As was mentioned above, the measured value of q_0, 4–5×10^{-10} coulombs/cm, is rather uncertain, but we know that q_0 is less than 8×10^{-10} coulombs/cm. Therefore the best way to determine whether there is agreement between the hypothesis and the experiments is to determine the value of q_0 from Eqs. (15) and (16), using the measured value of E_2, i.e. E'. But first we must determine the values of X for the regions of interest in the streamer.

5. *The field strength due to the excess positive space charge if no negative ions are formed*

From Fig. 11 it can be seen that if no negative ions are formed the excess positive space charge is $q_0[v_e/(v_e + v_t)]$. This value will now be represented by q_1. It is assumed that q_0 and v_t/v_e are constant, independent of the distance to the streamer tip. Then, of course, the field strength is determined by Eq. (13) and X equals E_1/q_1. In this case the value of b must be rather small—that is, of the same order as R—because almost all the positive ions are formed in the head of the streamer. The calculated values of E_1/q_1 are shown in Fig. 15 as a function of the distance to the streamer tip. It must be observed that we have a high field within a comparatively small region only and that the different values of R and b affect the field strength

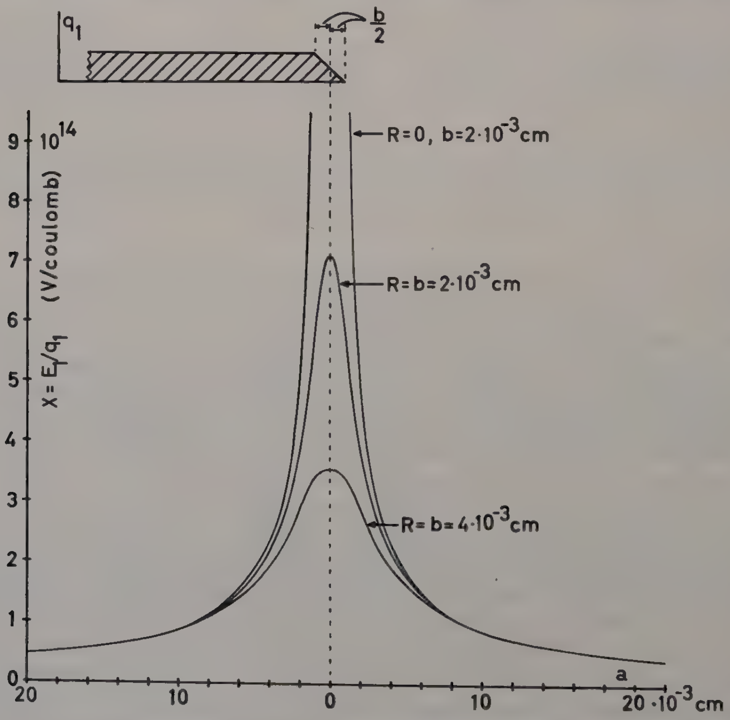

Fig. 15. The field strength due to a cylindrical space charge with the radius R.

only in this small region. At a distance of 6 × 10^{-3} cm from the streamer tip the value of E_1/q_1 is about the same as the maximum value of E_2/q_2 in Fig. 13.

At the tip of the streamer the field is high. Therefore, if negative ions are formed, their effect on the field strength must be rather unimportant in this region. But at a distance greater than about 6 × 10^{-3} cm from the tip the field due to the negative ions is of the same order as the field due to the positive ions, and the applied field is of considerable importance for the drift velocity of the electrons. Therefore, the largest value of X to be used in the determination of q_0 is about 1.0 × 10^{14} volts/coulomb. At a distance of 30 × 10^{-3} cm from the tip, where E_2/q_2 has its maximum (in oxygen) (Fig. 13), X is only 0.3 × 10^{14} volts/coulomb; at 40 × 10^{-3} cm it is 0.2 × 10^{14} volts/coulomb. At greater distances it decreases rather slowly.

6. *The value of q_0 and the critical field strength for oxygen*

Let us consider the case of oxygen. According to [21, 30] η is greatly dependent on the value of E/p, but let us assume that η equals $0.10 \cdot p$ cm^{-1} (p is measured in mm Hg), which corresponds to a value of E/p of about 10 volts/cm·mm Hg. The value of E/p can be expected to be of this order at atmospheric pressure, otherwise the removal of electrons from the streamer head would not be sufficient to maintain the necessary positive space charge. Consequently, η in Eq. (15) is substituted by 76 cm^{-1}.

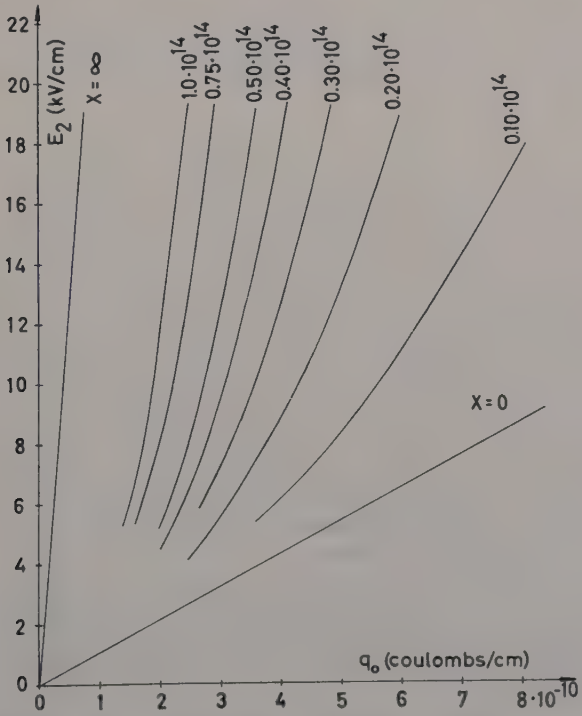

Fig. 16. The field strength due to the increase of the negative space charge when negative ions are formed. The field strength is shown as a function of q_0 for different values of X (cf. text).

We can now determine E_2 as a function of q_0 for different values of X. The chosen values of X are 1.00, 0.75, 0.50, 0.40, 0.30, 0.20 and 0.10×10^{14} volts/coulomb. These values of X are substituted in Eq. (16) and we can obtain v_t/v_e as a function of q_0. v_t/v_e is substituted in Eq. (15), and E_2 can be determined. The result of the calculation is shown in Fig. 16. The values of q_0 that give a field strength of the order of the critical field strength are $2\text{–}6 \times 10^{-10}$ coulombs/cm. For the sake of completeness graphs have also been drawn for $X = \infty$ and $X = 0$.

As is clear from Fig. 6, the critical field strength for oxygen is 14.2 kV/cm at atmospheric pressure. If no negative ions were formed in oxygen the value of E' would probably be about the same as for nitrogen, viz. about 3.0 kV/cm. Hence, the increase in the critical field strength due to the formation of negative ions must be 11 kV/cm. The lowest possible value of q_0 that can give rise to this field strength is 0.45×10^{-10} coulombs/cm. It must also be observed that this lowest value of q_0 is independent of the value of k in Eq. (16), because the graph for $X = \infty$ is the same for all values of k (with the exception, of course, of $k = 0$). In the same way we can conclude from the graph for $X = 0$ that q_0 cannot be much greater than 10×10^{-10} coulombs/cm; but this value of q_0 is not independent of k. It is lower the higher the value of k, and as the employed value of k (8.3×10^2 cm²/volt·sec) is lower than the real value for oxygen (10×10^2 cm²/volt·sec) [17], 10×10^{-10} coulombs/cm must be regarded as an upper limit. This is in good agreement with the experiments according to which q_0 must be less than 8×10^{-10} coulombs/cm.

As it has been necessary to make several approximations in the calculations, there is no point in trying to determine an exact value of q_0. The conclusion to be drawn from these calculations is that also quantitatively there seems to be a very good agreement between the results derived from the hypothesis and the values measured experimentally.

(*b*) The field strength necessary for maintaining the same space charge in the streamer if negative ions are formed as if no negative ions are formed

In the calculation above it was assumed that the drift velocity of the electrons was constant, independent of the distance to the streamer tip. As this is a rather crude approximation it is necessary to make a calculation also for the case where the velocity varies with the distance to the streamer tip. The following calculation will be made in quite another way than that made above. Only the first seven eqations are the same. It is to be observed that the only approximation that has been made in deriving Eq. (7) is that we have assumed η to be a constant, independent of the distance to the streamer tip. Eq. (7) gives the increase in the negative space charge owing to the formation of negative ions. The drift velocity of the electrons is denoted by $v_e'(l)$ if no negative ions are formed and by $v_e(l)$ if negative ions are formed. The expression for s is obtained from Eq. (3).

Now we shall calculate the increase of the applied field strength necessary to maintain the same space charge density if negative ions are formed as if no negative ions are formed. If the space charge density is the same in the two cases, the field strength owing to this must also be the same. The space charge is the same if in Eq. (7) $N(\eta) - N(\eta = 0) = 0$, i.e.

$$\frac{v_e'(l)}{v_e'(l) + v_t} - \frac{v_e(l)}{v_e(l) + v_t}\, e^{-\eta \cdot s} = 0. \tag{17}$$

As

$$\eta \cdot s = \eta \int_0^l \frac{v_e(l)}{v_e(l) + v_t}\, dl$$

is positive, $v_e(l)$ must be greater than $v_e'(l)$, i.e. the drift velocity of the electrons must be greater if negative ions are formed than if no negative ions are formed.

If $v_e(l)/[v_e(l) + v_t]$ is denoted by z and $v_e'(l)/[v_e'(l) + v_t]$ by y, Eq. (17) can be written (in logarithmic form)

$$\ln z - \eta \cdot \int_0^l z \cdot dl = \ln y. \tag{18}$$

As the field strength owing to the space charge is the same in both cases the increase in the drift velocity of the electrons when negative ions are formed must be caused by an increase in the applied field strength. The drift velocity of the electrons can be considered as proportional to the field strength [17]. Therefore the necessary increase in the applied field strength can be written $E_3 = (1/k) \,.\, (v_e(l) - v_e'(l))$, where k is the mobility. As we have no negative ions in the streamer when the drift velocity of the electrons is $v_e'(l)$, it is better to express E_3 as a function of $v_e'(l)$ and η than to express it as a function of $v_e(l)$ and η. Consequently, we must try to write Eq. (18) in another form. Fortunately, this is possible as the equation

$$z = \frac{y}{1 - \eta \int_0^l y \cdot dl} \tag{19}$$

is equivalent to Eq. (18). By using Eq. (19) we obtain

$$E_3 = \frac{1}{k} \cdot (v_e(l) - v_e'(l)) = \frac{v_t \cdot y \cdot \eta \int_0^l y \cdot dl}{k(1-y)\left(1 - y - \eta \int_0^l y \cdot dl\right)}. \tag{20}$$

In the general case y is a function of l and consequently E_3 is a function of l as well. $E_3 = \infty$ if $(1 - y - \eta \int_0^l y \cdot dl) = 0$, but this has no physical meaning, for the conditions on which Eq. (20) is built do not hold for very high values of the field strength. Eq. (20) is of interest only if the field strength E_3 is comparatively small, for it should be observed that the difference of the field strength in the streamer between the case where negative ions are formed and the case where no negative ions are formed is equal to E_3, if the space charges are the same in both cases. This means that the results derived from Eq. (20) should be compared with the critical field strength only for small values of η. However, this disadvantage is compensated by the fact that the radius and the charge produced per cm of the streamer do not appear in the equations.

The increase in the critical field strength owing to the formation of negative ions is equal to E_3 only if $E_3 \to 0$. Therefore, it is better to compare $(dE_3/d\eta)_{\eta=0}$ with

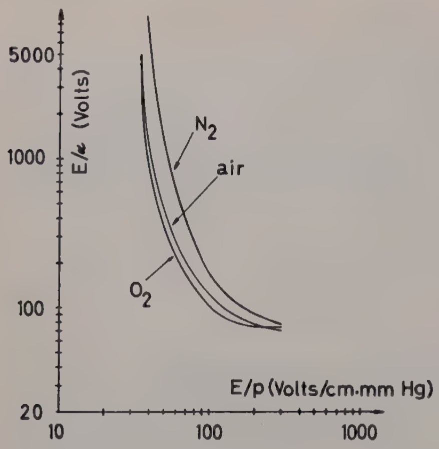

Fig. 17. E/α as a function of E/p for nitrogen, air, and oxygen.

the experimentally measured value of $(\Delta E'/\Delta\eta)$ that can be obtained from Fig. 6 than to make a comparison between E_3 and E'. From Eq. (20) we obtain

$$\frac{dE_3}{d\eta} = \frac{v_t \cdot y}{k} \cdot \frac{\int\limits_0^l y \cdot dl}{(1 - y - \eta \int\limits_0^l y \cdot dl)^2} \qquad (22\,\text{a})$$

and

$$\left(\frac{dE_3}{d\eta}\right)_{\eta=0} = \frac{v_t}{k} \cdot \frac{y}{(1-y)^2} \int\limits_0^l y \cdot dl. \qquad (22\,\text{b})$$

Unfortunately, we do not know $v_e'(l)$ as a function of l. However, as Eq. (22b) is so simple we can calculate $(dE_3/d\eta)_{\eta=0}$ for a number of cases where different function $v_e'(l)$ are used. Before this calculation can be made it is necessary to know something about what functions of $v_e'(l)$ should be used. It is for example improbable that $v_e'(l)$ would vary as $v_e'(l) = C \cdot l$, where C is a constant. A more probable example is $v_e'(l) = l_0/(l + A)$, where l_0 and A are constants. But what value of l_0 should be used? It is possible to ascertain this if we can estimate the energy that an electron must derive from the field in the streamer head.

As all the ionization by electrons must occur in the streamer head and just in front of it, the energy that an electron derives from the field in the streamer head must be at least some times that necessary to form an ion pair. This energy is greater than the ionization energy of a molecule, since the electron loses part of its energy by other processes than ionization. If α is known as a function of E/p it is possible to calculate the mean energy that an electron derives from the field for each ion pair formed. When an electron moves a distance dl, $\alpha \cdot dl$ ion pairs are formed. But the electron derives the energy $\mathrm{e} \cdot E \cdot dl$ from the field and consequently the energy $\mathrm{e} \cdot E/\alpha$ is used for producing one ion pair (here e is the electronic charge).

Several measurements of α have been made in oxygen, nitrogen and air [21–25]. In Fig. 17 the values of E/α calculated from these α-values are shown as a function

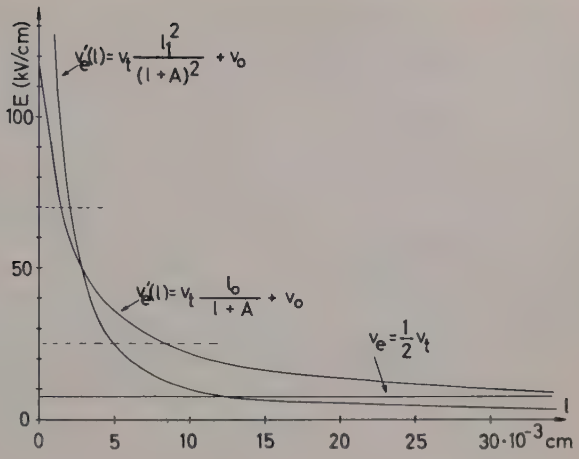

Fig. 18. The field strength used for calculating the values of $(dE_3/d\eta)_{\eta=0}$ shown in Fig. 19. The field strength is shown as a function of the distance to the streamer tip.

of E/p for O_2, N_2, and air. From Fig. 17 it is seen that an electron must derive about 100 electron volts from the field for each ion pair, if E/p is about 100 volts/cm mm Hg—that is, if the field strength is about 76 kV/cm at atmospheric pressure. Consequently, if an avalanche that is not too small is to be formed, the energy that the electron derives from the high field region in the streamer head must be at least 300–400 electron volts. The result of this discussion will now be used for choosing suitable functions of $v'_e(l)$.

If the drift velocity of the electrons is $v'_e(l)$, the field strength can be written $E = v'_e(l)/k$ [17]. The potential difference between the tip of the streamer ($l = 0$) and that part of the streamer where $l = L_0$ is

$$\Delta V = \int_0^{L_0} \frac{v'_e(l)}{k}\, dl. \tag{23}$$

As usual l is the distance to the streamer tip. As the ionization is very small if the field strength is lower than 25 kV/cm [21–25], the electron must derive about 400 electron volts in that part of the streamer where the field strength is greater than 25 kV/cm. Otherwise an avalanche cannot be formed.

Let us now assume that the drift velocity is given by

$$v'_e(l) = v_t \cdot \frac{l_0}{l + A} + v_0, \tag{24}$$

where l_0, A, and v_0 are constants. v_0 is the drift velocity at great distances from the streamer tip—that is, it is completely determined by the applied field. The constant A has been inserted only to give $v'_e(l)$ a reasonable value at the streamer tip ($l = 0$). The value of l_0 will be chosen so that the potential difference between the streamer tip and that part of the streamer where the field strength is 25 kV/cm is 400 volts. Let us suppose that the distance from the streamer tip to this part of the streamer is L_0. Then we obtain

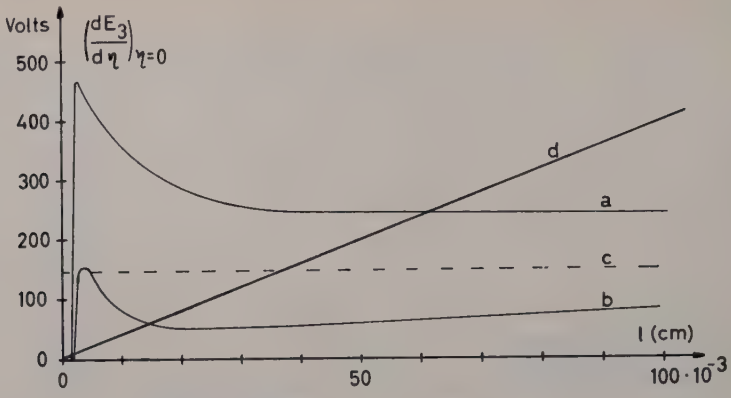

Fig. 19. $(dE_3/d\eta)_{\eta=0}$ as a function of l.

(*a*) $v_e'(l) = v_t \dfrac{l_0}{(l+A)} + v_0$,

(*b*) $v_e'(l) = v_t \dfrac{l_1^2}{(l+A)^2} + v_0$,

(*c*) experimentally,

(*d*) $v_e = \frac{1}{2} v_t = 6.5 \times 10^6$ cm/sec.

$$400 = \int_0^{L_0} \frac{v_t \dfrac{l_0}{l+A} + v_0}{k} \cdot dl, \tag{25a}$$

$$25\,000 = \frac{1}{k} \left[v_t \frac{l_0}{L_0 + A} + v_0 \right] \cdot \tag{25b}$$

If we substitute $k = 8.3 \times 10^2$ cm²/volt·sec [17], $v_t = 1.3 \times 10^7$ cm/sec, $A = 2 \times 10^{-3}$ cm and $v_0 = 2.5 \times 10^6$ cm/sec (the drift velocity if the field strength is 3 kV/cm), we obtain $L_0 = 8.4 \times 10^{-3}$ cm and $l_0 = 1.46 \times 10^{-2}$ cm. The greatest value of the field strength is in this case about 115 kV/cm (see Fig. 18).

As mentioned above, negative ions cannot be formed if the field strength is greater than about 70 kV/cm [29]. Consequently, no negative ions can be formed in the region just behind the tip of the streamer, but at distances from the streamer tip greater than 1.42×10^{-3} cm the field strength is lower than 70 kV/cm. Therefore, when we use Eq. (22b) to calculate $(dE_3/d\eta)_{\eta=0}$, the integration must be carried out not between the limits 0 and l but between $l = 1.42 \times 10^{-3}$ cm and l. The values of $(dE_3/d\eta)_{\eta=0}$ obtained in this way from Eq. (22b) are shown as a function of l (the distance to the streamer tip) in Fig. 19 (curve *a*).

If the same calculation is carried out with the assumption that

$$v_e'(l) = v_t \frac{l_1^2}{(l+A)^2} + v_0, \tag{26}$$

we obtain $l_1 = 8.28 \times 10^{-3}$ cm and find that the field strength is 70 kV/cm at a distance of 2.0×10^{-3} cm from the streamer tip (Fig. 18). The values of $(dE_3/d\eta)_{\eta=0}$ calculated in this way are also shown in Fig. 19 (curve *b*). These values are lower than those obtained above but they are of the same order of magnitude as the value determined experimentally. This can be obtained from Figs. 6 and 9 and is about 150 volts. It is shown by the dotted line in Fig. 19. The full line *d* shows the value, if $v_e'(l)$ is considered as a constant equal to $\frac{1}{2}v_t = 6.5 \times 10^6$ cm/sec. The two functions of $v_e'(l)$ used above for calculating $(dE_3/d\eta)_{\eta=0}$ have been chosen because they give a field strength in the streamer that decreases with the distance to the streamer tip in about the same way as the functions shown in Fig. 15. As is clear from Fig. 19 all the values of $(dE_3/d\eta)_{\eta=0}$ are of the same order of magnitude as that determined experimentally. However, we have only studied some special cases and as we do not know exactly how $v_e'(l)$ varies with l we can only conclude that it seems very probable that the formation of negative ions has a great influence on the critical field strength.

C. DISCUSSION AND CONCLUSIONS

1. The critical field strength for $O_2 + N_2$ mixtures

Even if it is accepted that the increase in the critical field strength with the percentage of oxygen is caused by the formation of negative ions, it is rather difficult to account for the details as the critical field strength can be measured much more exactly than many of the other quantities in Eq. (15). Besides, η is known only for nitrogen, air and oxygen. No measurements have been made in other mixtures of nitrogen and oxygen. We would expect η to be proportional to the partial pressure of oxygen, but this is not the case [21]. For nitrogen η/p equals zero, for air it is less than 0.005 (cm·mm Hg)$^{-1}$ and for oxygen it is 0.10 (cm·mm Hg)$^{-1}$. If there were a linear relation between η/p and the percentage of oxygen, η/p for air would be 0.02 (cm·mm Hg)$^{-1}$. As no measurements have been made in the range between air and oxygen we do not know if η varies linearly or if there are any irregularities.

As η/p has a low value for air, it is to be expected that it is low for all mixtures with lower percentage of oxygen than air. Hence, if streamers were formed in these mixtures the increase in the critical field strength with the percentage of oxygen would be small. The variation of the critical field strength with the percentage of oxygen is shown in Fig. 6. The uppermost graph gives the values at atmospheric pressure. For percentages of oxygen lower than 92 it is a straight line. We know that the critical field strength is 3.0 kV/cm for nitrogen (Fig. 7). However, the extension of the straight line does not intersect the ordinate at 3.0 kV/cm, but at a lower value. As it is improbable that if streamers were formed in this range the critical field strength would be lower than 3.0 kV/cm, the extension of the line can have no physical significance. It is to be expected that the critical field strength would be slightly larger than 3.0 kV/cm and vary somewhat like the dotted curve (Fig. 6). However, this divergence from the linear relation is explained in a simple manner by the hypothesis. As η increases slowly in the range 0–20 % oxygen, E_2 must also increase slowly, and consequently the increase of the critical field strength must be rather small. At higher values of the percentage of oxygen η must increase rather rapidly, and this is also the case with the critical field strength. If the percentage of oxygen is higher than 92, the critical field strength decreases somewhat;

but as we do not know how η varies with the percentage of oxygen, it is impossible to decide whether this is caused by a variation of η or by any other process.

If the percentage of oxygen in a $N_2 + O_2$ mixture is constant, E' is approximately proportional to the pressure of the gas mixture. Therefore, the variation of the critical field strength with the pressure (Fig. 5) agrees very well with the assumption that the increase of the critical field strength is caused by the formation of negative ions. Of course, we must observe that also other factors that can have a certain influence on the critical field strength are dependent on the pressure. However, as has been shown above, this influence must be rather small compared to that of the negative ions.

2. Conclusions

From the foregoing account it is evident that all but one of the processes that may give rise to the great variation of the critical field strength must be rejected in a qualitative study. In a more detailed quantitative study it was found that the negative ions formed in the streamer can be expected to give rise to a field strength of the same order as the critical field strength.

A complete explanation of the dead time of the positive point streamer corona can now be given. In Part I of this investigation it was shown that negative ions formed in the streamer prevent the streamer formation when they reach the positive point. As long as negative ions or electrons arrive at the point, no streamer can be formed. In this paper it is shown that the field strength necessary for the propagation of the streamer (the critical field strength) is dependent on the ability of the gas to form negative ions. The greater this ability is—that is, the larger η is—the greater the critical field strength.

Let us now consider the case of the corona hygrometer [1, 2]. By point-to-plane geometry the field strength decreases with increasing distance to the point, and consequently the streamers are shorter the larger the value of η. As η for air varies with the amount of water vapour [31]—that is, the greater the relative humidity (constant temp.) the greater the value of η—the streamers are shorter in air of high humidity than in air of low humidity. Consequently, the dead time is shorter and the number of streamers formed per second greater, the higher the relative humidity. Thus, the variation of η with the humidity of the air explains why the dead time t_0 (Eq. (4), Part I) is less at high values of the relative humidity than at low.

ACKNOWLEDGEMENTS

The author wishes to express his sincere thanks to Professor Sten von Friesen for his continued interest in this investigation. He is also very indebted to Docent Hellmuth Hertz for many valuable discussions. Thanks are due to Professor H. Raether, Hamburg, for his interest in the study.

The work has been financially supported by the Swedish Technical Research Council, which is gratefully acknowledged.

Department of Physics, University of Lund, Sweden, March, 1958.

REFERENCES

1. N. E. Andersson and C. H. Hertz, Z. angew. Phys. *7*, 361 (1955).
2. N. E. Andersson, Arkiv för Fysik, *13*, 399 (1958).
3. L. B. Loeb and W. Leigh, Phys. Rev. *51*, 149 (1937).

4. A. F. Kip, Phys. Rev. *54*, 139 (1938); Phys. Rev. *55*, 549 (1939).
5. L. B. Loeb and A. F. Kip, J. Appl. Phys. *10*, 142 (1939).
6. G. W. Trichel, Phys. Rev. *55*, 382 (1939).
7. K. E. Fitzsimmons, Phys. Rev. *61*, 175 (1942).
8. W. N. English, Phys. Rev. *74*, 170 (1948).
9. H. W. Bandal, Phys. Rev. *84*, 92 (1952).
10. G. L. Weissler, Phys. Rev. *63*, 96 (1943).
11. L. B. Loeb, J. Appl. Phys. *19*, 882 (1948).
12. M. R. Amin, J. Appl. Phys. *25*, 358 (1954).
13. L. B. Loeb, Phys. Rev. *94*, 227 (1954).
14. E. L. Huber, Phys. Rev. *97*, 267 (1955).
15. B. Murphy, Rep., Brit. Elect. Res. Assoc. L/T 329 (1956) (abstract).
16. H. Raether, Erg. ex. Naturwiss. *22*, 73 (1949).
17. A. von Engel, Ionized Gases. Oxford, 1955.
18. Handbuch der Physik XXI. Springer-Verlag, 1956.
19. K. Watanabe, J. Chem. Phys. *26*, 542 (1957).
20. B. F. Baker and J. T. Tate, Phys. Rev. *53*, 683 (1938).
21. M. A. Harrison and R. Geballe, Phys. Rev. *91*, 1 (1953).
22. M. A. Harrison, Phys. Rev. *105*, 366 (1957).
23. F. H. Sanders, Phys. Rev. *41*, 667 (1932); Phys. Rev. *44*, 1020 (1933).
24. K. Masch, Arch. Elektrotech. *26*, 589 (1932).
25. D. Q. Posin, Phys. Rev. *50*, 650 (1936).
26. J. T. Tate, P. T. Smith, and A. L. Vaughan, Phys. Rev. *48*, 525 (1935).
27. W. W. Lozier, Phys. Rev. *36*, 1417 (1930).
28. L. B. Loeb, Basic Processes of Gaseous Electronics. University of California Press, 1955.
29. L. B. Loeb, Phys. Rev. *48*, 684 (1935).
30. D. S. Burch and R. Geballe, Phys. Rev. *106*, 183 (1957).
31. A. M. Cravath, Phys. Rev. *33*, 605 (1929).

Tryckt den 23 april 1958

Uppsala 1958. Almqvist & Wiksells Boktryckeri AB